Generalized Implicit Definitions on Finite Structures

Stéphane Grumbach[⋆1] and **Zoé Lacroix**[⋆⋆2] and **Steven Lindell**[⋆⋆⋆3]

[1] University of Toronto and INRIA
[2] Université de Paris-Sud and INRIA
[3] Haverford College

Abstract. We propose a natural generalization of the concept of implicit definitions over finite structures, allowing non-determinism at an intermediate level of a (deterministic) definition. These generalized implicit definitions offer more expressive power than classical implicit definitions. Moreover, their expressive power can be characterized over unordered finite structures in terms of the complexity class NP $\cap$ co-NP. Finally, we investigate a subclass of these where the non-determinism is restricted to the choice of a unique relation with respect to an implicit linear order, and prove that it captures UP $\cap$ co-UP also over the class of all finite structures. These results shed some light on the expressive power of non-deterministic primitives.

1 Introduction

Let $\mathcal{K}$ be a class of structures over a vocabulary σ, and R a relation symbol not in σ. A first-order sentence $\varphi(R)$ over $\sigma \cup \{R\}$ *implicitly defines* a relation on $I \in \mathcal{K}$, if there is a unique relation R^I, such that $I \models \varphi(R^I)$. A k-ary relation R is *explicitly definable* on $I \in \mathcal{K}$ if there is a first-order formula $\varphi(\bar{x})$ over σ, with k free variables such that $R = \{\bar{a} \in \mathrm{I}^k : I \models \varphi(\bar{a})\}$. An explicit definition expresses a property of the tuples in the relation, whereas an implicit definition expresses a property of the relation itself.

Beth's definability theorem [Bet53] says that if a relation is implicitly definable over the class of *all* σ-structures, then it is explicitly definable. More generally, it is well known that this result fails when restricted to specific classes of models. In particular, this failure was illustrated by Gurevich for the class of finite structures [Gur84]. Implicit definitions over finite structures were then further investigated by Kolaitis, where their expressive power as definitions of

⋆ I.N.R.I.A. Rocquencourt BP 105, 78153 Le Chesnay, France – E-mail: stephane.grumbach@inria.fr – Work supported in part by an EC-NSF grant DEUS EX MACHINA, and an NSERC fellowship in Canada.

⋆⋆ L.R.I., Université de Paris Sud, 91405 Orsay, France – E-mail: zoe@lri.fr – Work done in the Verso Project at INRIA.

⋆⋆⋆ Department of Computer Science, Haverford College, Haverford, PA 19041-1392 – E-mail: slindell@haverford.edu – Work partially supported by NSF grant CCR-9403447 and the John C. Whitehead faculty research fund at Haverford College.

queries was studied [Kol90]. A sentence $\varphi(R)$ over $\sigma \cup \{R\}$ implicitly defines a *query* on $\mathcal{K}$, if for every structure I in $\mathcal{K}$, there is a unique relation R^I, such that $I \models \varphi(R^I)$.

More generally, Kolaitis introduced the class IMP of queries mapping finite structures over σ to some implicitly definable k-ary relation R. A k-ary query Q is definable in IMP on $\mathcal{K}$ if there exists a sentence $\varphi(R_1, ..., R_n)$ over $\sigma \cup \{R_1, ..., R_n\}$ where $R_1, \ldots, R_n$ are relation symbols not in σ and R_1 is of arity k, such that on any σ-structure $I \in \mathcal{K}$, there is a unique sequence of relations $\langle R_1^I, ..., R_n^I \rangle$, such that $I \models \varphi(R_1^I, ..., R_n^I)$ and $Q(I) = R_1^I$. It was shown in [Kol90] that fixpoint queries [CH82] can be expressed implicitly, as soon as the definition is based on *two* implicitly defined relations. Implicit definitions based on only one relation symbol provide less expressive power. On the other hand, it is easy to show that every implicit query can be expressed by only two implicitly defined relations, the output, R, and some other relation, S, by a first-order sentence $\varphi(R, S)$ over $\sigma \cup \{R, S\}$, such that $\varphi(R, S)$ implicitly defines R and S, that is for each finite σ-structure I,

$$I \models \exists! R \, \exists! S \; \varphi(R, S).$$

The relation S can be seen as an *intermediate relation* used as the "working area", necessary to get the full expressive power of IMP. The assumption that the intermediate relation is itself unique seems rather restrictive and unjustified. We propose a generalization of the concept of an implicit query, by allowing non-determinism at the level of the intermediate relation S, provided the output relation R is still deterministic. We do this by means of a sentence $\varphi(S)$ satisfied by all intermediate relations, and a formula $\psi(S, \bar{x})$ which is S-invariant with respect to φ defining the (unique) output relation explicitly. This relaxation of the strict uniqueness of classical implicit relations was first considered in [Lin87], where it was shown to be always equivalent to Δ_1^1 definability. We call this class of *generalized implicit queries*, GIMP.

This results in a strict increase in expressive power, i.e. IMP $\subset$ GIMP. As an example, the query true if the cardinality of the domain is even can be expressed in GIMP, while it is not definable in IMP. In fact, it is easy to verify directly that every PTIME query is definable in GIMP. Using the inductive definition of PTIME [Imm86], it suffices to first guess an order on the domain, and then implicitly specify the fixed-point (and its negation). For further details consult [Lin87].

The use of non-determinism to compute deterministic queries has also been studied by Abiteboul, Simon and Vianu [ASV90]. For a non-deterministic language $\mathcal{L}$, they introduce two different deterministic semantics, the *possibility* semantics, and the *certainty* semantics. In the *possibility* semantics, each formula of $\mathcal{L}$ defines the set of tuples satisfying the formula for at least one of the non-deterministic choices. In the *certainty* semantics, each formula of $\mathcal{L}$ defines the set of tuples satisfying the formula for all choices. Generalized implicit definitions are related to these semantics in the following way. Queries in GIMP are those such that on every finite σ-structure the *possibility* and *certainty* semantics coincide.

In this paper, we prove the general (non-boolean) case of the theorem in [Lin87].

Theorem 4.1 GIMP = NP $\cap$ co-NP, on the class of all finite structures.

We then go on to investigate in more detail the precise expressive power resulting from the non-determinism allowed in the intermediate relation. In GIMP, it is possible to "guess" a linear order on the domain of a finite structure, and it is well known that the order relation plays a fundamental role in the characterization of complexity classes below NP by logical formalisms [Var82, Imm86]. We consider a natural restriction, LIMP, of GIMP, where the non-determinism is restricted to the choice of some order relation on the finite domain, and prove our main theorem.

Theorem 5.2 LIMP = UP $\cap$ co-UP, on the class of all finite structures,

where UP denotes the class of queries computed by *unambiguous* Turing machines, i.e. NP machines with at most one accepting computation on every input [Val76].

This last theorem generalizes a similar result [Kol90] IMP = UP $\cap$ co-UP which applies only on the more restrictive class of all *ordered* structures. So the relative expressive power of various non-deterministic primitives is linked with difficult open problems in complexity theory.

2 Preliminaries

A *vocabulary* σ is a finite sequence $\{P_1, \ldots, P_s\}$ of relation symbols of fixed arities. A *σ-structure* I is a set I, called the *universe*, along with a mapping associating for all $i \in \{1, \ldots, s\}$ a relation P_i^I over I with the same arity as P_i. A σ-structure is finite if its universe I is a finite set. $\mathcal{K}$ denotes a class of σ-structures, $\mathcal{F}$ the class of all finite σ-structures, and $\mathcal{O}$ the class of finite ordered σ-structures. $\mathcal{L}$ is a logic language.

For k a non-negative integer, let a *k-ary query* (often called a *global relation*) Q on $\mathcal{K}$ be a mapping that associates to each σ-structure $I \in \mathcal{K}$ a k-ary relation $Q(I)$ on the universe I such that for all σ-structures I and J and isomorphisms f with $f(I) = J$, tuple $\langle a_1, \ldots, a_k \rangle \in Q(I)$ iff $\langle f(a_1), \ldots, f(a_k) \rangle \in Q(J)$. If $k = 0$, then Q is called a *boolean query*, because $Q^I \subseteq I^0 = \{\langle\,\rangle\}$ means Q is either $1 = \{\langle\,\rangle\}$ or $0 = \emptyset$. So Q determines the characteristic function of a subclass of $\mathcal{K}$ given by $\{I \in \mathcal{K} : Q(I) = 1\}$. The complement of a k-ary query Q is $\neg Q$ such that for each σ-structure I, $\neg Q(I) = \mathrm{I}^k - Q(I)$.

Definition 2.1 A k-ary query Q ($k \geq 0$) is *explicitly defined* in $\mathcal{L}$ on $\mathcal{K}$ if there is a formula of $\mathcal{L}$ $\varphi(x_1, \ldots, x_k)$ with k free variables such that for each $I \in \mathcal{K}$, $Q(I) = \{\bar{a} \in \mathrm{I}^k : I \models \varphi(\bar{a})\}$.

Generally, when a query is said to be definable in the logic $\mathcal{L}$, it means *explicitly* definable in $\mathcal{L}$. In the following, we often omit the term *explicit* when referring to queries (explicitly) definable in: FO (first-order logic); FP (fixed-point logic); Σ^1_1 (existential second-order logic); and $\mathrm{U}\Sigma^1_1$ (unique existential second-order logic).

Definition 2.2 A k-ary query Q ($k \geq 0$) is definable in Σ^1_1 if there exists a first-order formula $\varphi(S, x_1, \ldots, x_k)$ over $\sigma \cup \{S\}$ with k free variables such that for each $I \in \mathcal{K}$, $Q(I) = \{\bar{a} \in \mathrm{I}^k : I \models \exists S\varphi(S, a_1, \ldots, a_k)\}$.

A query Q is definable in Π^1_1 if $\neg Q$ is definable in Σ^1_1.

A query Q is definable in Δ^1_1 if both Q and $\neg Q$ are definable in Σ^1_1.

There is a strong correspondence between computational complexity classes and these logics. A logic $\mathcal{L}$ is said to capture a complexity class $\mathcal{C}$ if:

- for any formula φ of $\mathcal{L}$, the set $\mathrm{MOD}(\varphi)$ of the models of φ (the set of structures that satisfy φ) is recognizable in $\mathcal{C}$,
- for all class $\mathcal{K}$ of sets recognizable in $\mathcal{C}$ that is closed under isomorphisms, there exists a formula φ in $\mathcal{L}$ such that $\mathcal{K} = \mathrm{MOD}(\varphi)$.

Fagin's theorem [Fag74] shows that the logic Σ^1_1 captures the computational complexity class NP. Valiant considered a subclass of NP called UP, which denotes the class of queries computed by *unambiguous* Turing machines, i.e. NP machines with at most one accepting computation on every input [Val76]. The corresponding logic $\mathrm{U}\Sigma^1_1$, a subclass of Σ^1_1, was introduced by Kolaitis such that for any tuple that satisfies the formula there exists a unique relational witness [Kol90].

Definition 2.3 A k-ary query Q ($k \geq 0$) is definable in $\mathrm{U}\Sigma^1_1$ if there exists a first-order formula $\varphi(S, x_1, \ldots, x_k)$ over $\sigma \cup \{S\}$ with k free variables such that for each $I \in \mathcal{K}$, $Q(I) = \{\bar{a} \in \mathrm{I}^k : I \models \exists S\varphi(S, a_1, \ldots, a_k)\}$ and $I \models \forall\bar{x}[\exists S\varphi(S, x_1, \ldots, x_k) \rightarrow \exists! S\varphi(S, x_1, \ldots, x_k)]$ (where $\exists! S$ means "there is exactly one relation S").

A query Q is definable in $\mathrm{U}\Pi^1_1$ if $\neg Q$ is definable in $\mathrm{U}\Sigma^1_1$.

A query Q is definable in $\mathrm{U}\Delta^1_1$ if both Q and $\neg Q$ are definable in $\mathrm{U}\Sigma^1_1$.

Remark : The set Σ^1_1 is a recursively enumerable set of formulas. In contrast, $\mathrm{U}\Sigma^1_1$ is a co-recursively enumerable set, because the condition $I \models \forall\bar{x}\exists! S\varphi(S, \bar{x})$ is expressible in Π^1_1 : $I \models \forall\bar{x}\forall S\forall T(\varphi(S, \bar{x}) \wedge \varphi(T, \bar{x}) \rightarrow \forall\bar{y}(S\bar{y} \leftrightarrow T\bar{y}))$. In other words, a sentence $\exists S\varphi(S, \bar{x}) \notin \mathrm{U}\Sigma^1_1$ iff $\exists\bar{x}\exists S\exists T\varphi(S, \bar{x}) \wedge \varphi(T, \bar{x}) \wedge \exists\bar{y}(S\bar{y} \wedge \neg T\bar{y})$ is finitely satisfiable.

On the class $\mathcal{O}$, UP is characterized by the logic $\mathrm{U}\Sigma^1_1$. Notice that a built-in order is required to get the correspondence, because it is not possible to postulate a unique order in general.

3 Implicit Definability

An explicit definition of a global relation Q expresses a property of the tuples in Q, whereas an implicit definition expresses a property of the entire global relation Q. To illustrate this we present the notion of an individual global relation being implicitly definable, to be followed by the natural generalization to a sequence of such relations (as described in [Kol90]).

An individual global relation Q of arity k is said to be *implicitly defined* on a class of σ-structures $\mathcal{K}$ if there exists a first-order sentence $\varphi(S)$ over $\sigma \cup \{S\}$, where S is a new relation symbol of arity k, such that for each I in $\mathcal{K}$, $Q(I)$ is the unique relation that satisfies φ on I. The global implicit definition of Q over $\mathcal{K}$ is a k-ary query. Let us use IMP_1 to denote the class of individual implicitly definable queries.

Note that an explicitly defined (FO) query is also implicitly definable. Generally, the converse is not true. On the class of *all* σ-structures, Beth's theorem says that explicit and implicit definability coincide [Bet53]. However, this fails on $\mathcal{K} = \mathcal{F}$ as illustrated by Gurevich [Gur84].

Consider the following example of an IMP_1 query.

Example 3.1 Let σ be a vocabulary containing only one binary relation symbol $<$. The unary query Q consisting of the set of even elements with respect to $<$, if $<$ is a linear order on the structure, and of the empty set, otherwise, is implicitly defined by the first-order sentence $\varphi(S)$ where S is a new unary relation symbol.

$$\varphi(S) \equiv [\theta(<) \rightarrow \phi(S)] \wedge [\neg\theta(<) \rightarrow \forall x \neg Sx],$$

where $\theta(<)$ asserts that $<$ is a linear order and

$$\phi(S) \equiv \forall x[(\forall y\ \neg y < x) \rightarrow \neg Sx] \wedge \forall x \forall y\ [\neg Sx \leftrightarrow (\neg(\exists z\ x < z < y) \rightarrow Sy)]$$

expresses that S is the set of even elements with respect to the linear order $<$. More succinctly, if the domain is $\{1, \ldots, n\}$, $\phi(S)$ says $\neg S(1) \wedge \forall x[\neg S(x) \leftrightarrow S(x+1)]$. Q is not explicit since there is no first-order formula that defines the set of even elements on finite total orders (use 0/1 laws [Fag76], or Ehrenfeucht-Fraïssé games [Fra54, Ehr61]).

It is easy to see that IMP_1 is closed under complementation: If $\varphi(S)$ is an implicit definition of a query Q, then $\varphi'(T)$ obtained from $\varphi(S)$ by replacing each occurrence of $S\bar{u}$ by $\neg T\bar{u}$ is an implicit definition of $\neg Q$. But the disjunction (resp. conjunction) of two such implicit definitions is not necessarily implicit (unless they define the same query). Moreover, it is impossible to individually define an implicit *boolean* query without making it explicit.

This leads us to generalize the notion of an implicit definition of an individual relation to an implicit definition of a *sequence of relations*. A sequence of global relations $\langle R_1, \ldots, R_m \rangle$ is implicitly defined on a class of σ-structures $\mathcal{K}$ by the sentence $\varphi(S_1, \ldots, S_m)$, where each $S_i \notin \sigma$ is of the same arity as R_i, if for each I in $\mathcal{K}$, $\langle R_1, \ldots, R_m \rangle$ is the unique sequence of relations that satisfies φ on I. A query Q of arity k is implicitly definable in a sequence if it can be taken to be the first global relation in such a sequence. The class of all such queries over a class of structures $\mathcal{K}$ is called $\mathrm{IMP}_m(\mathcal{K})$ [Kol90], where m is the number of relations defined in the sequence. Letting IMP$= \bigcup \mathrm{IMP}_m$, we get that IMP is closed under all first-order operations: complementation, conjunction, disjunction, and quantification.

It is possible to reduce the sequence to just two relations. Let Q be the query implicitly defined in the sequence determined by a sentence $\varphi(S_1, \ldots, S_m)$ such that if $\langle S_1^I, \ldots, S_m^I\rangle$ is the unique sequence of relations that satisfies φ on a structure I, then $Q(I) = S_1^I$. Then there exists an equivalent implicit definition given by $\varphi'(S,T)$ obtained from $\varphi(S_1, \ldots, S_m)$, by replacing each occurrence of $S_1\bar{u}$ by $S\bar{u}$, and using T to encode the Cartesian product of $S_2, \ldots, S_m$. Since a sequence of two relations suffices, IMP $=$ IMP_2. The following example, which is an elaboration of an example in [Kol90], illustrates that IMP_1 is not as powerful.

Fact : The transitive-closure of a binary relation, TC, is not in IMP_1. Suppose that TC is implicitly definable in IMP_1, and defined by $\psi(S)$ involving a unique symbol of relation S of arity 2. Then ψ', obtained from $\psi(S)$ by replacing each occurrence of Sxy by $x = x \wedge y = y$, would be an explicit definition of connectivity (a graph is connected if its transitive closure is the Cartesian product of the domain). But connectivity is not first-order definable [Fag75].

Example 3.2 Let σ be a vocabulary consisting in one binary relation E. The binary query TC associating to a graph E its transitive closure has a definition $\varphi(S, \preceq)$ in IMP where S is of arity two and $\preceq$ of arity four. More precisely, $\varphi(S, \preceq)$ defines implicitly the sequence $\langle TC, \leq_\psi\rangle$ where $\psi(S,x,y) \equiv Exy \vee \exists z(Exz \wedge Szy)$ is the fixed-point formula defining TC and $\leq_\psi$ is similar to the *stage comparison preorder* defined in [Mos74]: for all binary tuples (a,b) and (a',b'), $(a,b) \leq_\psi (a',b')$ iff $d(a,b) \leq d(a',b')$, where $d(a,b)$ measures the distance from a to b, and is taken to be infinite is there is no path from a to b. This construction follows Kolaitis' one for positive fixed-point definitions [Kol90].

More generally, on the class $\mathcal{F}$, FP $\subset$ IMP. The inclusion was proved in [Lin87, Kol90]. Dawar, Hella and Kolaitis, showed that on the class $\mathcal{F}$, there exist implicit queries which are not fixed-point definable [DHK95]. But simple polynomial-time queries such as *Evenness* are not expressible in IMP. In fact, over $\mathcal{F}$, IMP is a complete co-recursively enumerable set of formulas. The problem to tell whether or not a first-order formula $\varphi(S)$ over $\sigma \cup \{S\}$, where $S \notin \sigma$ of arity k, is in IMP is not decidable [Kol90]. IMP is co-r.e. since for each sentence $\varphi(S)$, we can enumerate (up to isomorphism) all finite σ-structures I, and check if there is is no R^I such that $I \models \varphi(R^I)$ or if there are at least two different relations which satisfy $\varphi(S)$ on I.

More than the number of the implicitly defined relations in the sentence, it is the arity of the implicitly defined relation that has an impact on the expressive power. We prove that the full expressive power of implicit definitions in IMP can be obtained by only one implicitly defined relation associated with an explicit first-order transformation, instead of two implicitly defined relations.

Proposition 3.1 A k-ary query Q $(k \geq 0)$ is definable in IMP iff it has a definition of the form $\exists U\ \Psi(U) \wedge \varphi(U, \bar{x})$, where $\Psi(U)$ is a sentence in IMP_1 over $\sigma \cup \{U\}$, and $\varphi(U, \bar{x})$ is a formula with k free variables.

Proof: Let $\psi(S,T)$ be a sentence of IMP that defines Q. S and T are relation symbols of arity respectively k and n, that are not in σ. On every finite σ-structure I there exists a unique sequence of relations (S^I, T^I) such that $I \models \psi(S^I, T^I)$ and $S^I = Q(I)$. It is easy to encode the Cartesian product of S and T in a new relation symbol U of arity $(n+k)$. There is a critic situation when on a structure I, the relation T^I is the empty set. In fact we want to make sure that when U^I is the empty set then S^I is the empty set.

Consider the following definition $[\psi(S,T) \wedge \exists\bar{x}T\bar{x}] \vee \eta(S)$ where η is obtained from $\psi(S,T)$ by replacing each occurrence of $T\bar{u}$ by $\bar{u} \neq \bar{u}$ (that forces T to be the empty set). $[\psi(S,T) \wedge \exists\bar{x}T\bar{x}] \vee \eta(S)$ is a definition in IMP of Q. For each σ-structure I, $I \models \psi(S^I, T^I)$ then $I \models [\psi(S^I, T^I) \wedge \exists\bar{x}T^I\bar{x}] \vee \eta(S^I)$. $\langle S^I, T^I \rangle$ is the unique sequence of relations that satisfy the latter definition.

Let $\psi'(U)$ be the sentence obtained from $[\psi(S,T) \wedge \exists\bar{x}T\bar{x}]$ replacing each occurrence of $S\bar{u}$ (resp. $T\bar{u}$) by $\exists\bar{v}U\bar{u}\bar{v}$ (resp. $\exists\bar{v}U\bar{v}\bar{u}$) in $[\psi(S,T) \wedge \exists xT\bar{x}]$ and $\eta'(U)$ the one obtained from $\eta(S)$ replacing each occurrence of $S\bar{u}$ by $\forall\bar{v}U\bar{u}\bar{v}$ in $\eta(S)$. The disjunction $\psi'(U) \vee \eta'(U)$ is still exclusive. Then, let U^I be a $(k+n)$-ary relation on a σ-structure I such that $I \models \psi'(U^I) \vee \eta'(U^I)$. $\Psi(U) \equiv_{Df} \psi'(U) \vee \eta'(U) \in$ IMP since $\psi(S,T) \in$ IMP. More precisely, we have:

- if U^I is not empty, then
  - $I \models \eta'(U^I)$ implies $U^I = S^I \times \mathrm{I}^n$ and $I \models \psi(S^I, \emptyset)$ then $S^I \neq \emptyset$ and $T^I = \emptyset$,
  - $I \models \psi'(U^I)$ implies $U^I = S^I \times T^I$ and $I \models \psi(S^I, T^I)$ then S^I, $T^I \neq \emptyset$,
- if U^I is empty, then
  - $I \models \eta'(U^I)$ implies $U^I = \emptyset \times \mathrm{I}^n$ and $I \models \psi(\emptyset, \emptyset)$ then $S^I = T^I = \emptyset$,
  - $I \models \psi'(U^I)$ implies $U^I = \emptyset \times T^I$ and $I \models \psi(\emptyset, T^I)$ then $S^I = \emptyset$ and $T^I \neq \emptyset$.

Then $Q(I) = \{\bar{a} : I \models \exists U\Psi(U) \wedge \exists\bar{y}U\bar{a}\bar{y}\}$. Q is defined by $\Psi(U) \wedge \exists\bar{y}U\bar{x}\bar{y}$.

Conversely, if Q is defined by the formula $\Psi(U) \wedge \varphi(U, \bar{x})$ where Ψ is implicit, it is clear that $\psi(S,U) \equiv \Psi(U) \wedge \forall\bar{x}\ (S\bar{x} \leftrightarrow \varphi(U, \bar{x}))$ is a definition of Q in IMP. □

In the following, this normal form motivates our generalized implicit definitions.

4 Generalized Implicit Definitions

In this section, we extend the notion of an implicit definition. Instead of requiring a sentence to have a unique solution, we only require the uniqueness of solutions "modulo" another formula. Given a sentence $\psi(S)$ and a formula $\varphi(S, \bar{x})$, we require that on each structure, $\psi(S)$ always has at least one solution S^I, and that for any such solution, $\varphi(S^I, \bar{x})$ is unique. In a sense, S is being used as a non-deterministic "working area" for the purpose of defining a deterministic query. We call these *generalized* implicit definitions, and the class of queries they define, GIMP.

A definition of a k-ary query Q, $k \geq 0$, in GIMP is split into two successive steps:

(i) *non-deterministic*: the choice of a *solution* S^I that satisfies the sentence $\psi(S)$,
(ii) *deterministic*: the evaluation of the formula $\varphi(S, \bar{x})$ with k free variables at S^I.

We say that $\varphi(S, \bar{x})$ mod $\psi(S)$ defines a generalized implicit query if for every finite σ-structure, there is a solution which makes $\psi(S^I)$ true, and for each such solution $\varphi(S^I, \bar{x})$ determines the same relation.

Notice that although non-determinism is involved in defining the semantics of a query in GIMP, each query in GIMP has a deterministic semantics. The use of non-determinism to compute deterministic queries has been studied in [ASV90, Sch90, SZ90, GSZ95]. For a non-deterministic logical language $\mathcal{L}$, Abiteboul, Simon and Vianu introduce two different deterministic semantics, the *possibility* semantics, and the *certainty* semantics [ASV90]. In the *possibility* semantics, each formula of $\mathcal{L}$ defines the set of tuples satisfying that formula for at least one of the non-deterministic choices. That is, the semantics is defined as the union of the possible non-deterministic outputs of the formula. In the *certainty* semantics, each formula of $\mathcal{L}$ defines the set of tuples satisfying that formula for every non-deterministic choice. That is, the semantics is defined as the intersection of the possible non-deterministic outputs of the formula.

Generalized implicit definitions are related to these semantics in the following way. $\varphi(S, \bar{x})$ mod $\psi(S)$ defines a GIMP query iff the possibility semantics, $\{\bar{a} \in \mathrm{I}^k : I \models \exists S \psi(S) \wedge \varphi(S, \bar{a})\}$, and the certainty semantics, $\{\bar{a} \in \mathrm{I}^k : I \models \forall S(\psi(S) \rightarrow \varphi(S, \bar{a}))\}$, coincide for every finite σ-structure I. This leads to a new *Possible-is-Certain* deterministic semantics defined as follows.

Definition 4.1 For a non-deterministic language $\mathcal{L}$, a formula of $\mathcal{L}$ has the *Possible-is-Certain* semantics if the set of tuples satisfying the formula for at least one of the non-deterministic choices coincides with the set of tuples satisfying the formula for all non-deterministic choices.

However, it is important to note that not every pair admits a Possible-is-Certain semantics, and that GIMP is precisely the queries defined by pairs which admit such a semantics.

More formally, we define GIMP queries as follows.

Definition 4.2 A GIMP k-ary query Q over a class $\mathcal{K}$ of σ-structures, $S \notin \sigma$, is given by $\varphi(S, \bar{x})$ mod $\psi(S)$ where $\psi(S)$ is a sentence such that

(i) $\models_{\mathcal{K}} \exists S \psi(S)$

and $\varphi(S,\bar{x})$ is a formula with k free variables such that

(*ii*) $\psi(S) \models_{\mathcal{K}} \varphi(S,\bar{x}) \leftrightarrow Q(\bar{x})$.

Remark : Condition (*ii*) implies the S-invariance of $\varphi(S,\bar{x})$ with respect to $\psi(S)$, that is:
(*ii'*) $\psi(S), \psi(S') \models_{\mathcal{K}} \varphi(S,\bar{x}) \leftrightarrow \varphi(S',\bar{x})$.

Example 4.1 The *Evenness* query (true on a finite structure $\mathcal{B}$ iff the cardinality of $\mathcal{B}$ is even) is in GIMP. Let S be a relational symbol of arity 3 that is not in the vocabulary. The formula $\psi(S)$ will define the Cartesian product of a (strict) linear order and the set of even numbered elements with respect to it.

$$\begin{aligned}
\psi(S) \equiv\ & \forall x \forall y\ [\exists u Sxyu \vee x = y \vee \exists u Syxu] && (1)\\
& \wedge\ \forall x \forall y \forall u\ [Sxyu \rightarrow (\forall v \neg Syxv \wedge x \neq y)] && (2)\\
& \wedge\ \forall x \forall y \forall z \forall u \forall v\ [(Sxyu \wedge Syzv) \rightarrow \exists w Sxzw] && (3)\\
& \wedge\ \forall x\ [x = Min \rightarrow \forall y \forall z \neg Syzx] && (4)\\
& \wedge\ \forall x \forall y \forall u \forall v\ [(Sxyu \wedge v = Suc(u)) \rightarrow \forall z \forall t \neg Sztv] && (5)\\
& \wedge\ \forall x \forall y \forall u \forall v\ [(\neg Sxyu \wedge \exists w Sxyw \wedge v = Suc(u)) \rightarrow Sxyv] && (6)
\end{aligned}$$

where $x = Min \equiv \forall y \forall u\ \neg Syxu$ and

$$\begin{aligned}
v = Suc(u) \equiv \exists u\ Sxyu \wedge \forall z \forall u\ [Sxzu \rightarrow (\exists v Syzv \vee z = y)]\\
\wedge\ \forall z \forall u\ [Szyu \rightarrow (\exists v Szxv \vee z = x)].
\end{aligned}$$

The first three expressions say that the projection of the first two components is a linear order: (1) says that $<$ is total; (2) says that $<$ is anti-symmetric; and (3) says that $<$ is transitive. The next three expressions define the set of $EVEN$ elements with respect to the order induced on the domain $\{1,\ldots,n\}$ by the two first components: (4) says $1 \notin EVEN$; (5) says $x \in EVEN \rightarrow x+1 \notin EVEN$; and (6) says roughly that $x-1 \notin EVEN \rightarrow x \in EVEN$. Finally,

$$\forall x [\forall y \forall u \neg Sxyu \rightarrow \exists y \exists z Syzx] \bmod \psi(S)$$

which says that maximal element is in $EVEN$ is a definition of *Evenness* in GIMP.

It is easy to see that GIMP generalizes IMP_1 by choosing $\varphi(S,\bar{x}) \equiv S\bar{x}$. GIMP generalizes IMP since a definition in normal form $\psi(U) \wedge \varphi(U,\bar{x})$ of a query in IMP introduced in section 3 gives $\varphi(U,\bar{x}) \bmod \psi(U)$ in GIMP. Moreover, GIMP is closed under complementation, union, intersection, projection. The expressive power of GIMP admits a precise characterization in terms of complexity classes. We prove the following result.

Theorem 4.1 On the class $\mathcal{F}$ of all finite structures, GIMP = NP $\cap$ co-NP.

Proof : We first show the easy direction. Consider a query Q in GIMP, given by $\varphi(S,\bar{x})$ mod $\psi(S)$. Observe that on each finite σ-structure I,

$$Q(I) = \{\bar{a} \in \mathrm{I}^k : I \models \exists S\ \psi(S) \wedge \varphi(S,\bar{a})\} = \{\bar{a} \in \mathrm{I}^k : I \models \forall S(\psi(S) \rightarrow \varphi(S,\bar{a}))\}.$$

Hence, Q is included in both Σ^1_1 and Π^1_1. Therefore GIMP $\subseteq \Delta^1_1$. By Fagin's result [Fag74], NP is characterized by Σ^1_1 on finite structures, so the queries computable in NP $\cap$ co-NP are exactly those that have a Δ^1_1 definition. In fact, our proof will show that over any class of structures, GIMP is precisely Δ^1_1.

Conversely, let Q be a k-ary query definable in Δ^1_1. Without loss of generality, we can assume that there exist formulas $\exists S\ \phi(S,\bar{x})$ and $\exists T\ \theta(T,\bar{x})$, where ϕ and θ are first-order formulas with relations S and T of the same arity n, respectively defining Q and $\neg Q$ in Σ^1_1.

Let I be a finite σ-structure with domain I. For all $\bar{a} \in \mathrm{I}^k$, $\bar{a} \in Q(I)$ (respectively $\bar{a} \notin Q(I)$) iff there exists a relation $S_{\bar{a}}$ (respectively $T_{\bar{a}}$) such that $I \models \phi(S_{\bar{a}},\bar{a})$ (respectively $I \models \theta(T_{\bar{a}},\bar{a})$). Therefore, $I \models \forall\bar{x}\ [\exists S\ \phi(S,\bar{x}) \vee \exists T\ \theta(T,\bar{x})]$. Since the relation symbols S and T do not belong to σ, and are of the same arity, we can identify them to obtain $I \models \forall\bar{x}\ \exists S\ [\phi(S,\bar{x}) \vee \theta(S,\bar{x})]$.

Let S' be a new relation symbol of arity $(n+k)$, that intuitively encodes the skolemized dependency of S relatively to $\bar{x}$, interpreted by one of the sets $\{(\bar{a},\bar{b}) : \bar{a} \in Q(I)$ and $\bar{b} \in S_{\bar{a}}$, or $\bar{a} \notin Q(I)$ and $\bar{b} \in T_{\bar{a}}\}$. If we let $\psi(S')$ be the sentence $\forall\bar{x}[\phi'(S',\bar{x}) \vee \theta'(S',\bar{x})]$, where $\phi'(S',\bar{x})$ (resp. $\theta'(S',\bar{x})$) is obtained from $\phi(S,\bar{x})$ (resp. $\theta(S,\bar{x})$) by replacing each occurrence of $S(\bar{u})$ by $S'(\bar{x},\bar{u})$, then we can see that $I \models \exists S' \psi(S')$.

To conclude we claim that $\phi'(S',\bar{x})$ mod $\psi(S')$ defines Q in GIMP. Let I be a finite σ-structure and S_0 any relation of arity $(n+k)$ such that $I \models \psi(S_0)$. We will show that $I \models \phi'(S_0,\bar{a}) \Leftrightarrow \bar{a} \in Q(I)$. Indeed, $I \models \phi'(S_0,\bar{a}) \Rightarrow I \models \phi(\{\bar{b} : (\bar{a},\bar{b}) \in S_0\},\bar{a}) \Rightarrow \bar{a} \in Q(I)$ by the Σ^1_1 definition of Q. For the other direction, $I \not\models \phi'(S_0,\bar{a}) \Rightarrow I \models \theta'(S_0,\bar{a}) \Rightarrow I \models \theta(\{\bar{b} : (\bar{a},\bar{b}) \in S_0\},\bar{a}) \Rightarrow \bar{a} \notin Q(I)$ by the Σ^1_1 definition of $\neg Q$. The proof is now complete. □

Like IMP or Δ^1_1, GIMP is not a recursively enumerable class of queries. The question whether or not $\varphi(S,\bar{x})$ mod $\psi(S)$ is a correct definition in GIMP is clearly co-recursively enumerable. And indeed it is co-recursively enumerable complete since the validity problem of first-order sentences over finite structures can be reduced to it. For each first-order sentence θ over a vocabulary σ containing some binary relation symbol, we consider $\neg Sx$ mod $(\theta \wedge \forall x Sx) \vee (\neg\theta \wedge \exists x Sx)$ which is a correct implicit definition in GIMP on the class of finite structures (of cardinality at least 2) iff θ is valid. Trakhtenbrot's theorem asserts that the set of first-order sentences that are finitely valid is a complete co-recursively enumerable set [Tra50].

The question of whether there exists a recursively enumerable logic for GIMP is connected with the existence of a complete problem for NP $\cap$ co-NP. Gurevich [Gur88] showed that if there exists a logic that captures NP $\cap$ co-NP then NP $\cap$ co-NP has a complete problem with respect to polynomial time reducibility. Since our correspondence is constructive, a recursively enumerable logic for GIMP would testify to the existence of a complete problem for NP $\cap$ co-NP. Gurevich conjectures that if that were the case, something drastic would happen (like NP $\cap$ co-NP = P).

Proposition 4.2 Let $\psi(S)$ be a FO definition and $\mathcal{K}$ a class a finite σ-structures such that $\mathcal{K} \models \exists S\psi(S)$. Let $\mathcal{Q}$ be a k-ary query which has a definition in GIMP over $\sigma \cup \{S\}$.

If $\mathcal{Q}$ is S-invariant mod $\psi(S)$ on $\mathcal{K}$, then $\mathcal{Q}$ has a definition in GIMP over σ.

5 Generalized Implicit Definitions with Order

The generalized implicit definitions of GIMP proposed in Section 4 give a logical characterization of NP $\cap$ co-NP over finite structures. In this section, we examine what happens if we restrict the general non-determinism to the choice of a total order, while retaining the classical (unique) implicit definitions for any other further implicitly defined relations. We'll call this LIMP, a restriction of GIMP and a generalization of IMP, where the choice consists of a linear order $\prec$ over the domain of the finite structure and a unique relation S on $\prec$. More formally, we define LIMP queries as follows.

Definition 5.1 A k-ary query Q over the class $\mathcal{F}$ of finite σ-structures, with $\prec, S \notin \sigma$, is in LIMP if it can be defined by $\varphi(\prec, S, \bar{x})$ mod $\psi(\prec, S)$ where $\psi(\prec, S)$ is a sentence such that for each total linear order $\prec$ on I $\in \mathcal{F}$ there exists a unique S_0 such that

(*i*) $I \models \psi(\prec, S_0)$,

and $\varphi(\prec, S, \bar{x})$ is a formula with k free variables satisfying the following for each finite σ-structure I, for each pair $(\prec, S)$ satisfying $\psi(\prec, S)$ on I,

(*ii*) $I \models \varphi(\prec, S, \bar{x}) \leftrightarrow Q(\bar{x})$.

It is easy to see that LIMP $\subseteq$ GIMP by encoding together the order $\prec$ and the relation S into a new relation symbol U. The *Evenness* query has a definition in LIMP since its non-deterministic part consists of the choice of a linear order $\prec$ and the set of the even elements with respect to $\prec$ (that is unique).

Example 5.1 The *Evenness* query, true on a finite structure $\mathcal{B}$ iff the cardinality of $\mathcal{B}$ is even, has a definition in LIMP. Let S be a relational symbol of arity 1 that is not in the vocabulary. The formula $\psi(\prec, S)$ defines the set S of even elements with respect to the chosen order $\prec$.

$$\begin{aligned}\psi(\prec, S) \equiv \forall x \; [(\forall y \neg y \prec x) \rightarrow \neg Sx] \\ \wedge \forall x \; \forall y \; [Sx \leftrightarrow \; [\neg(\exists z \; x \prec z \prec y) \rightarrow \neg Sy]]\end{aligned}$$

Then $\forall x[\forall y \, \neg x \prec y \rightarrow Sx]$ mod $\psi(\prec, S)$ is a definition of *Evenness* in LIMP.

Since a definition in LIMP consists of the non-deterministic choice of a linear order, followed by an order-invariant part, it is natural to consider a one-one map from the class of queries $\mathcal{Q}$ over finite structures to the class of queries $\mathcal{Q}^*$ over finite ordered structures that are *order-invariant*. For each finite σ-structure I of size n, let $I_1^*, \ldots, I_{n!}^*$ be the $n!$ extensions of I over $\sigma \cup \{<\}$. For each finite $\sigma \cup \{<\}$-structure I^*, let I be its restriction to σ, i.e. the finite σ-structure I with the same domain I and the same interpretation of the symbols in σ.

For any k-ary query Q over the class of finite σ-structures, there exists a k-ary Q^* over finite $\sigma \cup \{<\}$-structures where $<$ is interpreted by a linear order, such that, for each finite σ-structure I of size n, $Q(I) = Q^*(I_1^*) = \cdots = Q^*(I_{n!}^*)$. It is clear that such a Q^* is order-invariant. Conversely, for each query Q^* on finite $\sigma \cup \{<\}$-structures that is *order-invariant* there is a query Q over finite σ-structures such that for all finite $\sigma \cup \{<\}$-structures I^*, $Q(I) = Q^*(I^*)$.

An interesting example of an order invariant query that is not first-order definable without the order relation is presented in ([AHV94], Exercise 17.27, page 462). The presence of an order has an important impact on definability because it rigidifies a finite structure. The decision problem of whether a first-order formula $\varphi(\prec, S, \bar{x})$ mod $\psi(\prec, S)$, defines a semantically correct query in LIMP over finite σ-structures is undecidable. This is proved by an easy reduction from the validity of first-order sentences on finite structures. Let α be a first-order sentence over σ (containing some binary relation symbol), then consider $\varphi(\prec, x) \equiv (\neg\alpha \wedge \forall y \neg y \prec x) \wedge (\alpha \wedge x = x)$. $\varphi(\prec, x)$ mod $\prec$ is in LIMP iff $\models_{\mathcal{F}} \alpha$. Furthermore, there is a first-order $\sigma \cup \{<\}$-sentence φ such that the decision problem whether φ is order invariant on a σ-structure I is co-NP complete [Gur88].

If $\mathcal{Q}$ is a class of queries on finite ordered structures, then let $\mathrm{Inv}_<(\mathcal{Q})$ be the class of order-invariant queries in $\mathcal{Q}$. The following lemma says that an order-invariant query Q^* is in LIMP iff it is computable in UP $\cap$ co-UP on finite ordered structures.

Lemma 5.1 On the class $\mathcal{O}$ of finite ordered structures,

$$\mathrm{Inv}_<(\mathrm{LIMP}) = \mathrm{Inv}_<(\mathrm{U}\Delta_1^1) = \mathrm{Inv}_<(\mathrm{UP} \cap \text{co-UP}).$$

Proof: We first prove that $\mathrm{Inv}_<(\mathrm{LIMP}) = \mathrm{Inv}_<(\mathrm{U}\Delta_1^1)$ on the class of finite structures ordered by $<$. Consider a k-ary query Q^* in $\mathrm{Inv}_<(\mathrm{LIMP})$ defined by $\varphi(\prec, S, \bar{x})$ mod $\psi(\prec, S)$ over $\sigma \cup \{<\}$. The Σ_1^1 formula $\exists S \psi(<, S) \wedge \varphi(<, S, \bar{x})$, where $\psi(<, S)$ and $\varphi(<, S, \bar{x})$ are obtained from $\psi(\prec, S)$ and $\varphi(\prec, S, \bar{x})$ by replacing each occurrence of $\prec$ by $<$, also defines Q^* on the class of $\sigma \cup \{<\}$-structures ordered by $<$. By definition of LIMP, we know that for any $\sigma \cup \{<\}$-structure I^*, there exists a unique relation S_0^* such that $I^* \models \psi(<, S_0^*)$. Hence, for all $\bar{a} \in \mathrm{I}^{*k}$, $I^* \models \exists! S \psi(<, S) \wedge \varphi(<, S, \bar{a})$. Therefore, Q^* has a definition in $\mathrm{U}\Sigma_1^1$. Since LIMP is closed under complementation, Q^* is definable in $\mathrm{U}\Delta_1^1$. Recall that Q^* was order-invariant, so $Q^* \in \mathrm{Inv}_<(\mathrm{U}\Delta_1^1)$.

In the other direction, let Q^* be a k-ary query in $\mathrm{Inv}_<(\mathrm{U}\Delta_1^1)$. So Q^* has a definition in $\mathrm{U}\Delta_1^1$ that does not depend on the order. From [Kol90], we know that IMP = UΔ_1^1 on any class of structures. So Q^* has a definition in IMP. Since LIMP is a generalization of IMP, it follows that $\mathrm{Inv}_<(\mathrm{IMP}) \subseteq \mathrm{Inv}_<(\mathrm{LIMP})$. Again, since Q^* is order-invariant, $Q^* \in \mathrm{Inv}_<(\mathrm{LIMP})$.

The second equality $\mathrm{Inv}_<(\mathrm{U}\Delta_1^1) = \mathrm{Inv}_<(\mathrm{UP} \cap \text{co-UP})$ comes from the fact that on finite ordered structures, UP=UΣ_1^1 [Kol90]. It follows that UP $\cap$ co-UP = UΣ_1^1 $\cap$ co-UΣ_1^1, and hence that $\mathrm{Inv}_<(\mathrm{U}\Delta_1^1) = \mathrm{Inv}_<(\mathrm{UP} \cap \text{co-UP})$. □

The following theorem shows more generally that LIMP also characterizes a complexity class on finite structures.

Theorem 5.2 On the class $\mathcal{F}$ of all finite structures, LIMP = UP $\cap$ co-UP.

Proof: Using the previous definitions, we first show that a query Q on finite structures is definable in LIMP iff its (order-invariant) extension Q^* on finite ordered structures is definable in LIMP. The following are equivalent: (i) $Q \in$ LIMP over σ, and (ii) $Q^* \in \mathrm{Inv}_<(\mathrm{LIMP})$ over $\sigma \cup \{<\}$.

$(i) \Rightarrow (ii)$ is immediate.

$(ii) \Rightarrow (i)$ Let Q^* be an order-invariant query defined by $\varphi(\prec, S, \bar{x})$ mod $\psi(\prec, S)$ over $\sigma \cup \{<\}$. Its restriction Q has a definition $\varphi'(\prec, S, \bar{x})$ mod $\psi'(\prec, S)$ in LIMP over σ obtained from $\varphi(\prec, S, \bar{x})$ mod $\psi(\prec, S)$ by replacing each occurrence of $<$ by $\prec$.

Next we show that an order-invariant query Q^* is computable in UP $\cap$ co-UP iff its restriction Q is in UP $\cap$ co-UP. The following are equivalent: *(iii)* $Q \in$ UP $\cap$ co-UP over σ, and *(iv)* $Q^* \in \mathrm{Inv}_<(\mathrm{UP} \cap \text{co-UP})$ over $\sigma \cup \{<\}$.

(iii)$\Rightarrow$*(iv)* is immediate.

(iv)$\Rightarrow$*(iii)* Let Q^* be a query in $\mathrm{Inv}_<(\mathrm{UP} \cap \text{co-UP})$ over $\sigma \cup \{<\}$. Let $\mathcal{M}^*$ be an unambiguous Turing machine that computes Q^*. Let $\mathcal{M}$ be a machine whose input is a linear encoding of a σ-structure I that first uses the order of entrance of the elements of I to determine an order and then follows exactly the same transitions as $\mathcal{M}^*$. $\mathcal{M}$ is unambiguous (the encoding of the order is deterministic and the following computation is done by $\mathcal{M}^*$) and computes Q. The same method gives an unambiguous machine that computes $\neg Q$. It follows that $Q \in$ UP $\cap$ co-UP over σ.

The result now follows from Lemma 5.1. □

6 Conclusion

We have studied two natural extensions of first-order implicit definitions, and show that they characterize well known complexity classes, even over *unordered* finite structures. These extensions are based on the ability to non-deterministically choose an (intermediate) relation R satisfying a sentence $\phi(R)$, and use it to (deterministically) compute a query, independent of the particular choice of R. The first extension GIMP, in which an arbitrary relation is non-deterministically chosen, captures NP $\cap$ co-NP. The second extension LIMP, in which only an order relation on the domain is non-deterministically chosen, captures UP $\cap$ co-UP.

It might seem that the non-deterministic choice of an order relation should be enough to capture the full expressive power of GIMP (since it rigidifies the structure), and that maybe the definitions of LIMP could be seen as a normal form for GIMP queries. Nevertheless, it follows from the results presented here that the non-determinism can be reduced to the choice of an order relation if and only if UP $\cap$ co-UP = NP $\cap$ co-NP.

Acknowledgments : The authors wish to thank Phokion Kolaitis, Richard Lassaigne, Michel de Rougemont and Victor Vianu for their suggestions and helpful discussions.

References

[AG89] M. Ajtai and Y. Gurevich. Datalog vs. first-order logic. In *30th IEEE Symp. on Foundations of Computer Science*, pages 142–146, 1989.

[AHV94] S. Abiteboul, R. Hull, and V. Vianu. *Foundations of Databases*. Addison-Wesley, 1994.

[ASV90] S. Abiteboul, E. Simon, and V. Vianu. Non-deterministic languages to express deterministic transformations. In *Proc. 9th ACM Symp. on Principles of Database Systems*, 1990.

[Bet53] E. W. Beth. On Padoa's method in the theory of definition. *Indag. Math.*, 15:330–339, 1953.

[CH82] A. Chandra and D. Harel. Structure and Complexity of Relational Queries. *Journal of Computer and System Sciences*, 25(1):99–128, Aug. 1982.

[DHK95] A. Dawar, L. Hella and Ph.G. Kolaitis. Implicit definability and infinitary logic in finite model theory. Proc. 22nd International Colloquium on Automata, Languages and Programming - ICALP 95, Lecture Notes in Computer Science, Springer-Verlag, to appear.

[Ehr61] A. Ehrenfeucht. An application of games to the completness problem for formalized theories. Fund. Math. 49: 129–141, 1961.

[Fag74] R. Fagin. Generalized first-order spectra and polynomial-time recognizable sets. *Complexity of Computations, SIAM-AMS Proceedings 7*, pages 43–73, 1974.

[Fag75] R. Fagin. Monadic generalized spectra. Zeitschrift für Math. Logik 21: 89–96, 1975.

[Fag76] R. Fagin. Probabilities on finite models. Journal of Symbolic Logic 41: 50–58, 1976.

[Fra54] R. Fraïsse. Sur quelques classifications des systèmes de relations. Publications Scientifiques de l'Université d'Alger, Série A 1: 35–182, 1954.

[Gur84] Y. Gurevich. *Computation and Proof Theory (M. Richter et al. Ed.)*, chapter Towards Logic tailored for Computational Complexity, pages 175–216. Lecture Notes in Mathematics, 1104, 1984.

[Gur88] Y. Gurevich. *Current Trends in Theoretical Computer Science, E. Borger Ed.*, chapter Logic and the Challenge of Computer Science, pages 1–57. Computer Science Press, 1988.

[GSZ95] S. Greco and D. Saccà and C. Zaniolo. DATALOG Queries with Stratified Negation and Choice: from P to D^P. In *Proc. ICDT'95*, Springer Lecture Notes in Computer Science 893, 82–96, 1995.

[Imm86] N. Immerman. Relational queries computable in polynomial time. *Inf. and Control*, 68:86–104, 1986.

[Kol90] P. Kolaitis. Implicit definability on finite structures and unambiguous computations. In *Proc. 5th Symp. of Logic in Computer Science*, 1990.

[Lin87] S. Lindell. The Logical Complexity of Queries on Unordered Graphs. Ph.D. thesis, University of California at Los Angeles, 1987.

[Mos74] Y. N. Moschovakis. *Elementary Induction on Abstract Structures.* North Holland, 1974.

[Tra50] B. A. Trakhtenbrot. Impossibility of an algorithm for the decision problem in finite classes. *Doklady Akademii Nauk SSSR*, 70:569–572, 1950.

[Sch90] J.S. Schlipf. The Expressive Powers of the Logic Programming Semantics. In *Proc. 9th ACM Symp. on Principles of Database Systems*, 196–204, 1990.

[SZ90] D. Saccà and C. Zaniolo. Stable Models and Non-Determinism in Logic Programs with Negation. In *Proc. 9th ACM Symp. on Principles of Database Systems*, 205–217, 1990.

[Val76] L. Valiant. Relative complexity of checking and evaluating. *Information Processing*, 5:20–23, 1976.

[Var82] M. Vardi. The complexity of relational query languages. In *Proc. 14th ACM Symp. on Theory of Computing*, pages 137–146, 1982.