

# Datalog Evaluation

Serge Abiteboul

INRIA

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## READ CHAPTER 13

lots of research in the late 80'th

top-down or bottom-up evaluation

direct evaluation vs. compilation into a more efficient program

no product

some influence on logic programming

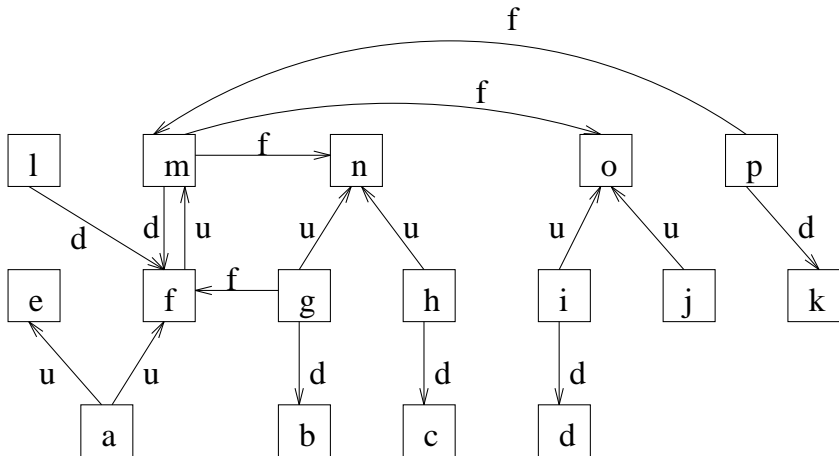
## HERE

- 1 semi-naive bottom-up evaluation
- 2 top-down : QSQ
- 3 bottom-up : Magic

# Reverse-Same-Generation

$rsg(x, y) \leftarrow flat(x, y)$   
 $rsg(x, y) \leftarrow up(x, x1), rsg(y1, x1), down(y1, y)$

<i>up</i>	<i>flat</i>	<i>down</i>
<i>a e</i>	<i>g f</i>	<i>l f</i>
<i>a f</i>	<i>m n</i>	<i>m f</i>
<i>f m</i>	<i>m o</i>	<i>g b</i>
<i>g n</i>	<i>p m</i>	<i>h c</i>
<i>h n</i>		<i>i d</i>
<i>i o</i>		<i>p k</i>
<i>j o</i>		



# Naive algorithm

level 0 :  $\emptyset$   
level 1 :  $\{(g, f), (m, n), (m, o), (p, m)\}$   
level 2 :  $\{\text{level 1}\} \cup \{(a, b), (h, f), (i, f), (j, f), (f, k)\}$   
level 3 :  $\{\text{level 2}\} \cup \{(a, c), (a, d)\}$   
level 4 :  $\{\text{level 3}\}$

$rsg := \emptyset$

repeat

$rsg := rsg \cup flat \cup \pi_{16}(\sigma_{2=4}(\sigma_{3=5}(up \times rsg \times down)))$

until *fixpoint*

$rsg^{i+1} = rsg^i \cup flat \cup \pi_{16}(\sigma_{2=4}(\sigma_{3=5}(up \times rsg^i \times down)))$

redundant computation

each layer recomputes all elements of the previous layer

monotonicity

## Semi-naive

Focus on the new facts generated at each level

RSG'

$$\Delta_{rsg}^1(x, y) \leftarrow flat(x, y)$$

$$\Delta_{rsg}^{i+1}(x, y) \leftarrow up(x, x1), \Delta_{rsg}^i(y1, x1), down(y1, y)$$

not recursive

not a datalog program

for each input I,

$$rsg^{i+1} - rsg^i \subseteq \delta_{rsg}^{i+1} \subseteq rsg^{i+1}$$

$$RSG(I)(rsg) = \cup_{1 \leq i} (\delta_{rsg}^i).$$

less redundancy

## An improvement

$$\delta_{rsg}^{i+1} \neq rsg^{i+1} - rsg^i$$

e.g.,  $(g, f) \in \delta_{rsg}^2$ , not in  $rsg^2 - rsg^1$

use  $rsg^i - rsg^{i-1}$  instead of place of  $\Delta_{rsg}^i$  in the second “rule” of RSG'

Non linear rules : e.g., ancestor

$$anc(x, y) \leftarrow par(x, y)$$

$$anc(x, y) \leftarrow anc(x, z), anc(z, y)$$

semi-naive evaluation

$$\Delta_{anc}^1(x, y) \leftarrow par(x, y)$$

$$\Delta_{anc}^{i+1}(x, y) \leftarrow \Delta_{anc}^i(x, z), anc^i(z, y)$$

$$\Delta_{anc}^{i+1}(x, y) \leftarrow anc^i(x, z), \Delta_{anc}^i(z, y)$$

still some redundancy : use

$$temp^{i+1}(x, y) \leftarrow \Delta_{anc}^i(x, z), anc^{i-1}(z, y)$$

$$temp^{i+1}(x, y) \leftarrow anc^i(x, z), \Delta_{anc}^i(z, y),$$

## Top-down technique : Query-Sub-Query

Program + query

$$rsg(x, y) \leftarrow flat(x, y)$$
$$rsg(x, y) \leftarrow up(x, x1), rsg(y1, x1), down(y1, y)$$
$$query(y) \leftarrow rsg(a, y)$$

Focus on relevant data

avoid deriving unnecessary tuples

A is relevant if there is a proof tree for *query* in which the fact occurs

We will do that (not perfectly) using **four ingredients**

- (a) SLD-resolution but set-at-a-time using relational algebra
- (b) start with query constants and “push” constants from goals to subgoals (in the style of pushing selections into joins)
- (c) Use “sideways information passing” to pass constant binding information from one atom to the next in subgoals
- (d) Use an efficient global flow-of-control strategy



# Technique

adornment and adorned rules  
supplementary relations and QSQ templates  
the kernel of the technique  
global control strategies

# Adornment

in  $\leftarrow rsg(a, y)$ , we have a binding for the first argument of  $rsg$

we denote it  $rsg^{bf}$

later on, we need  $rsg^{fb}$

based on the evaluation of subqueries  $(rsg^{fb}, \{\langle e \rangle, \langle f \rangle\})$   
in general  $(R^\gamma, J)$  where

- 1  $R$  is a predicate
- 2  $\gamma$  an adornment
- 3  $J$  provides bindings
- 4 completion for  $t$  in  $J$
- 5 answer of the subquery : set of completions

## Sideways information passing

from relational optimization

- evaluation of joins
- $P(a,v) \text{ join } Q(b,w,x) \text{ join } R(v,w,y)$
- evaluate first  $P(a,v) \rightarrow$  obtains some  $v$ 's
- evaluate then  $R(v,w,y)$  restricted by  $v$ 's  $\rightarrow$  obtains some  $w$ 's
- finally evaluate  $Q(b,w,x)$  restricted by  $w$ 's

### Adorned rule

$$R(x, y, z) \leftarrow R_1(x, u, v), R_2(u, w, w, z), R_3(v, w, y, a)$$

a subquery involving  $R^{bfb}$  with left-to-right evaluation  
(reorder if necessary)

$$R^{bfb}(x, y, z) \leftarrow R_1^{bff}(x, u, v), R_2^{bffb}(u, w, w, z), R_3^{bbfb}(v, w, y, a)$$

given head adornment and an ordering of body, algo for  
adorning

## Supplementary relations and QSQ templates

Data structure to store information during evaluation  
 $n + 1$  supplementary relations for a rule with  $n$  atoms

$$R^{bfb}(x, y, z) \leftarrow R_1^{bff}(x, u, v), R_2^{bffb}(u, w, w, z), R_3^{bbfb}(v, w, y, a)$$
$$\begin{array}{cccc} \uparrow & \uparrow & \uparrow & \uparrow \\ sup_0[x, z] & sup_1[x, z, u, v] & sup_2[x, z, v, w] & sup_3[x, y, z] \end{array}$$

variables serve as attribute names in the supplementary relations  
QSQ template for the adorned rule :  $sup_0, sup_1, sup_2, sup_3$

## Kernel of QSQ evaluation

input : program + query

construct adorned rules for the query :

$$(1) \quad rsg^{bf}(x, y) \leftarrow flat(x, y)$$

$$(2) \quad rsg^{fb}(x, y) \leftarrow flat(x, y)$$

$$(3) \quad rsg^{bf}(x, y) \leftarrow up(x, x1), rsg^{fb}(y1, x1), down(y1, y)$$

$$(4) \quad rsg^{fb}(x, y) \leftarrow down(y1, y), rsg^{bf}(y1, x1), up(x, x1)$$

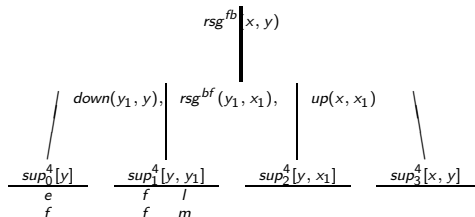
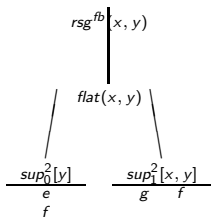
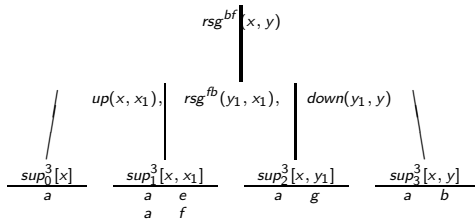
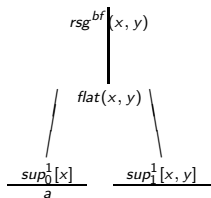
(Note the ordering to pass bindings in (4))

construct QSQ template for each adorned rule  $i$   
and the supplementary relation variables  $sup_j^i$

construct also : for each idb  $R$  and adornment  $\gamma$

- 1 the variable  $ans\_R^\gamma$  of arity  $arity(R)$
- 2 the variable  $input\_R^\gamma$  of arity  $bound(R, \gamma)$

initialization : use the query



input\_rsg<sup>bf</sup>  
a

input\_rsg<sup>fb</sup>  
e  
f

ans\_rsg<sup>bf</sup>  
a b

ans\_rsg<sup>fb</sup>  
g f

## Kinds of steps

A : from  $input\_R^\gamma$  to  $sup_0^i$

move new tuples in  $input\_R^\gamma$  to 0<sup>th</sup> supplementary relations

B : from  $sup_j^i$  to  $sup_{j+1}^i$

Pass new tuples from one supplementary relation to the next

C : from  $ans_R^\gamma$  to  $sup_j^i$

Use new *idb* tuples (from  $ans\_R^\gamma$ ) to generate new supplementary relation tuple

D : from  $sup_n^i$  to  $ans\_R^\gamma$

Process tuples in the final supplementary relation of a rule

# Example

$\langle a \rangle$	<i>into</i>	$input\_rsg^{bf}$	init
$\langle a \rangle$		$sup_0^1, sup_0^3$	A
$\langle a, e \rangle, \langle a, f \rangle$		$sup_1^3$	B
$\langle e \rangle, \langle f \rangle$		$input\_rsg^{fb}$	B
$\langle g, f \rangle$		$ans\_rsg^{fb}$	B,D and 2nd rule
$\langle a, b \rangle$		$ans\_rsg^{bf}$	C,B,D and 3rd rule



## Global control strategies

basic one : apply steps (A) through (D) until a fixpoint is reached

QSQR (query-subquery-recursive)

in step (B) : pass from supplementary relation  $sup_{j-1}^i$  across an *idb* predicate  $R^\gamma$  to supplementary relation  $sup_j^i$

introduce new tuples into  $sup_j^i$  and into  $input\_R^\gamma$

perform a recursive call to determine the  $R^\gamma$ -values corresponding to the new tuples added to  $input\_R^\gamma$ , before considering the new tuples placed into  $sup_j^i$

see algorithm in the book

# Merci