Different Kinds of Comparisons Between Fuzzy Conceptual Graphs

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Abstract. In the context of a microbiological application, our study proposes to extend the Conceptual Graph Model in order to allow one: (i) to represent imprecise data and queries that include preferences, by using fuzzy sets (from fuzzy set theory) in concept vertices, in order to describe either an imprecise concept type or an imprecise referent; (ii) to query a conceptual graph that may include imprecise data (factual graph) using a conceptual graph that may include preferences (query graph). This is performed in two steps: firstly by extending the projection operation to fuzzy concepts, secondly by defining a comparison operation characterised by two matching degrees: the possibility degree of matching and the necessity degree of matching between two graphs, and particularly between a query graph and a factual graph.

1 Introduction

Our research project is part of a national program that aims at building a tool for microbial risk analysis in food products. This tool is based on an information system which is composed of two parts: a relational database and a conceptual graph knowledge base. The latter is used to store information that was not expected when the schema of the database was designed, and is useful nevertheless. As modifying the schema of the database is quite an expensive operation, the conceptual graph model [11] appears as a flexible way of representing complementary information until the relational database is updated. More precisely the formalization of [9] is used in our application.

Both bases are queried simultaneously by a unified querying system. This part of our work has already been presented in [2]. Both have to deal with the following two specificities. (i) Some of the data are imprecise, like data whose precision is limited by the measuring techniques. For instance, when using a method able to detect bacteria beyond a given concentration threshold (e.g. 10^2 cells per gramme), not detecting any bacterium means that their concentration is below this threshold, which is an imprecise value denoted "$< 10^2$ cells/g". (ii) The bases are incomplete, as they will never contain information about all possible food products and all possible pathogenic germs. Those two characteristics led us to propose, firstly the handling of imprecise values, and secondly the expression of different levels of preferences in the user's selection criteria so as to allow flexible querying.

In the bibliography concerning databases, the fuzzy set framework has been shown to be a sound scientific way of modelling both flexible queries [1] and imprecise values by means of possibility distributions [10]. Besides, the introduction of the fuzzy set theory into the conceptual graph model has been initially studied by Morton [8] and then extended by several works such as [14, 4]. Morton distinguished different kinds of fuzziness. In particular, linguistic fuzziness concerns metric attribute concepts, which can have either a precise measure or a label that stands for a crisp or fuzzy subset of what is called the "universe of discourse". In [14], linguistic fuzziness is proposed for non-metric attributes, and fuzzy relation vertices are introduced, by associating a certainty degree with relations.

In these studies, the notion of fuzzy marker is not clearly introduced: what is fuzzy is the measure associated with a given concept, and the definition domain of this measure seems external to the support. The notion of fuzzy concept type is not handled. Our work introduces the notions of fuzzy marker and fuzzy type. In both cases, fuzziness is represented homogeneously, by means of a normalized fuzzy set. The "universe of discourse" is clearly defined as part of the support of the conceptual graph model, and metric and non-metric concepts are not distinguished as they are treated in the same way. We do not propose fuzzy relations, as in our context fuzziness concerns the data and not the way they are linked. However, our study about fuzzy types could be applied to any hierarchy and thus to the relation type set.

In [4], the notion of conjunctive fuzzy type is proposed, which is a conjunction of types associated with the same individual marker (with different fuzzy truth values). In our approach to fuzzy types, we do not question the unicity of an individual marker's type: a fuzzy type represents a disjunction of possible types (with different possibility degrees).

We presented preliminary studies of our approach in [3, 13, 12], where basic principles were formalized, including the treatment of numerical values and the introduction of fuzzy markers and fuzzy types. In this paper, we focus on different ways of querying imprecise data using fuzzy queries. A first way is the extension of the projection operation to fuzzy conceptual graphs, which is an all-or-nothing process. A second way is the use of comparison degrees of the fuzzy set theory, which allows one to perform fuzzy querying. In order to introduce these two comparisons, we went deeper into the notion of fuzzy type, by distinguishing a fuzzy type in intention (or simply, a fuzzy type) from its associated developed form called fuzzy type in extension.

In Section 2, we recall our choices for representing fuzzy values in the conceptual graph model, that is, fuzzy markers and fuzzy types, and we introduce the notion of fuzzy type in extension. In Section 3, we propose an extension of the projection operation to handle fuzzy values. In Section 4, we introduce a more flexible way of comparing fuzzy conceptual graphs, using two comparison degrees of the fuzzy set theory, the possibility degree of matching and the necessity degree of matching.

2 Fuzzy Values in the Conceptual Graph Model

Information of the application stored in conceptual graphs (factual graphs or query graphs) may be represented in two ways: (i) as individual markers, as it is the case for numerical values (30, 50, etc.) or for bacterial strains (E-3, A-86, etc.); (ii) as concept types, as it is the case for substrates (*Milk*, *Beef*, etc.). In both cases, we must be able to represent them as fuzzy information, as explained in Part 2.1. We thus have introduced the representation of fuzzy values concerning both markers, presented in Part 2.2, and concept types, presented in Part 2.3. As mentioned in the introduction, in this paper we propose a more thorough definition of fuzzy types, compared to [3, 13, 12].

2.1 Preliminary Notions: Fuzzy Sets

In our application we need firstly to be able to represent imprecise data, secondly to express preferences in queries. To perform this we use the fuzzy set theory [15].

Definition 1 *a* **fuzzy set** *A on a domain X is defined by a membership function μ_A from X to [0, 1] that associates the degree to which x belongs to A with each element x of X .*

The domain X may be continuous or discrete. These two cases are illustrated by the examples given in Figure 1. The fuzzy set *MyMilkProductPreferences* is also denoted : *1/Whole milk + 0.5/Half skim milk*, which indicates the degree associated with each element.

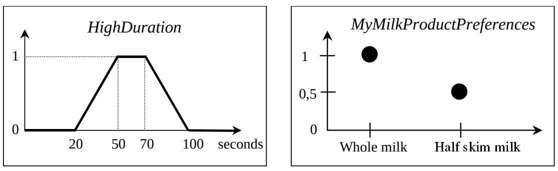

Fig. 1. Fuzzy sets *HighDuration* and *MyMilkProductPreferences*

A fuzzy set may be interpreted in two ways:

1. in a query, it expresses preferences on the domain of a selection criterion. For example, the fuzzy set *HighDuration* in Figure 1 may be interpreted as a preference concerning the required value of the criterion *Duration*: a duration between 50 and 70 seconds is fully satisfactory; values outside this interval may also be acceptable, but with smaller preference degrees;
2. in a datum, it describes an imprecise datum represented by a possibility distribution. For example, the fuzzy set *MyMilkProductPreferences* may be interpreted as an imprecise datum if the kind of milk that was used in the experiment is not clearly known: it is very likely to be whole milk, but half skim milk is not excluded.

In the following we recall the definitions of fuzzy markers and fuzzy types and we introduce the new notion of fuzzy type in extension. We refer to the conceptual graph model of [11], and more precisely to the formalisation of [9].

2.2 Fuzzy markers

Definition 2 *A* **fuzzy marker** m_f *of concept type* t *is a fuzzy set defined on the set of individual markers* I*. It takes values between 0 and 1 for every individual marker that conforms*[1] *to* t*, and 0 elsewhere.*

A fuzzy marker represents a disjunction of individual markers that conform to type t, weighted by a coefficient between 0 and 1.

Remark 1 *A "classic" individual marker* m *of type* t *can be considered as a particular fuzzy marker: its membership function associates the value 1 with* m*, and the value 0 with the rest of* I*.*

*The generic marker * can be considered as a particular fuzzy marker of type* t*: its membership function associates the value 1 with any element that conforms to* t*, and 0 with the rest of* I*.*

Definition 3 *A* **concept with a fuzzy marker** *is a concept vertex whose label is a pair* (t, m_f)*, where* m_f *is a fuzzy marker of concept type* t*.*

The conceptual graph represented in Figure 2 includes a concept with a fuzzy marker, of type *NumericalValue*.

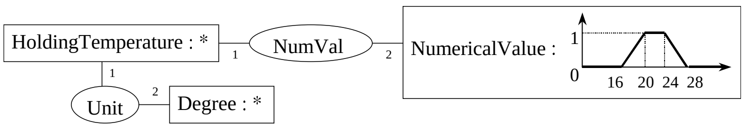

Fig. 2. An example of a concept with a fuzzy marker

2.3 Fuzzy types

Definition 4 *A* **fuzzy type** t_f *is a fuzzy set defined on a subset* D_{t_f} *of elements of the concept type set* T_C*.*

For example, the fuzzy set *MyMilkProductPreferences* represented in Figure 1 is a fuzzy type defined on a subset of the concept type set.

Remark 2 *A "classic" concept type* t *can be considered as a particular fuzzy type: its membership function is defined on one element* $\{t\}$ *and takes the value 1 for this element.*

[1] The set of individual markers that conform to t is $\{m \in I \mid \tau(m) \leq t\}$, where τ is the application from I to T_C, defined in the support, that associates a minimum concept type with each individual marker.

We can note that no restriction has been imposed concerning the concept types that compose the definition domain of a fuzzy type. In particular, the user may associate a given degree d with a type t and another degree d' with a subtype t' of this type. $d' \leq d$ represents a semantic of restriction for t' compared to t, whereas $d' \geq d$ represents a semantic of reinforcement for t' compared to t. For instance, if the user is particularly interested in skim milk because he studies the properties of low fat products, but also wants to retrieve complementary information about other kinds of milk, he will be able to express his preferences using e.g. the following fuzzy type: *1/Skim milk + 0.5/Milk*.

However the information *1/Skim milk* and *0.5/Milk* may sound contradictory. Indeed, the criterion *0.5/Milk* seems to include skim milk, which is a kind of milk, though skim milk has the degree 1. In fact, a fuzzy type defined on a subset of T_C implicitly gives information about the rest of T_C. In our example, if the user's preferences are those expressed by the fuzzy type *1/Skim milk + 0.5/Milk*, we can deduce in addition that he is not interested in vegetable.

This observation led us to introduce the notion of **fuzzy type in extension**, which is the developed form of the fuzzy type defined in Definition 4, which we now call **fuzzy type in intention**. A fuzzy type in extension is defined on the whole concept type set T_C.

Definition 5 *Let t_{fi} be a fuzzy type in intention defined on a subset D_i of T_C. Its membership function is denoted μ_i. The* **fuzzy type in extension** *t_{fe} associated with t_{fi} is defined as follows.*

The membership function of t_{fe} is denoted μ_e. For each element $t \in T_C$:

- *if there exists, at least, one element $u \in D_i$ such that $u \geq t$, then we distinguish two cases:*
  - *if there exists one single smallest[2] element $u \in D_i$ such that $u \geq t$, then $\mu_e(t) = \mu_i(u)$*
    The interpretation is the following:
    - *if the fuzzy set expresses preferences, then being interested in a given type u with a degree d implies being interested in all the types t more specific than u with the same degree of preference.*
    - *if the fuzzy set expresses an imprecise datum, then declaring that a given type u has a degree of possibility d implies that, for all t which are more specific than u, their degree of possibility is d.*
  - *otherwise, we call $u_1, u_2, \ldots, u_n$ the smallest elements of D_i such that $\forall k \in \{1, 2, \ldots, n\}\ u_k \geq t$. These elements are not comparable[3]. We distinguish two cases:*
    - *if the fuzzy set expresses preferences, then the degree of preference associated with t must be at least equal to the degree of preference associated with each element of the list $u_1, u_2, \ldots, u_n$, and we define $\mu_e(t) = max_{k \in \{1,2,\ldots,n\}} \mu_i(u_k)$*

[2] with the meaning of the relation **a kind of** that partially orders T_C

[3] using the partial order induced by the relation **a kind of**

* *if the fuzzy set expresses an imprecise datum, then t cannot have a degree of possibility greater than each of those associated with* $u_1, u_2, \ldots, u_n$*, and we define* $\mu_e(t) = min_{k\in\{1,2,\ldots,n\}}\mu_i(u_k)$

– *otherwise* $\mu_e(t) = 0$

Figure 3 shows the fuzzy type in extension associated with the fuzzy type in intention *0.8/Milk + 1/Whole milk + 0.3/Evaporated milk* on a part of the concept type set. We assume that the fuzzy set expresses an imprecise datum.

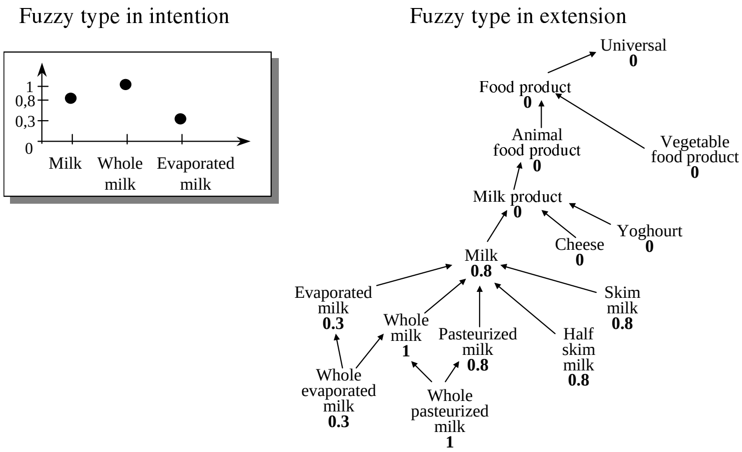

Fig. 3. An example of a fuzzy type in extension

In the fuzzy type in intention of Figure 3, the user has associated the degree 1 with *Whole milk* but only 0.3 with *Evaporated milk*. The minimum of these two degrees is thus associated with their common subtype *Whole evaporated milk* in the fuzzy type in extension, because we assume that the fuzzy type expresses an imprecise datum. If, on the contrary, it expressed preferences, the degree associated with *Whole evaporated milk* would be 1 instead of 0.3.

On the other hand, there is no ambiguity about *Whole pasteurized milk* because nothing has been specified about *Pasteurized milk* in the fuzzy type in intention. Therefore, *Whole pasteurized milk* has the degree associated with *Whole milk*, that is 1, and not 0.8 which is associated with the more general type *Milk*.

Definition 6 *A* **concept with a fuzzy type** *is a concept vertex whose label is a pair* (t_f, m)*, where* t_f *is a fuzzy type and* m *is the generic marker* *.

For instance, let us suppose that the user's preferences concerning the substrate are *MyMilkProductPreferences* represented in Figure 1. In conceptual graph terms, this substrate corresponds to the concept type *Whole milk* with the degree 1, or the concept type *Half skim milk* with the degree 0.5, which is represented by the concept with a fuzzy type of Figure 4.

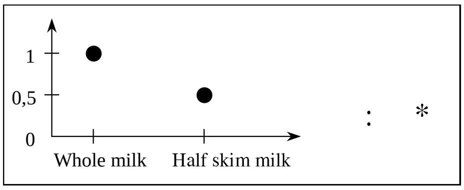

Fig. 4. An example of a concept with a fuzzy type

Definition 7 *In a concept with a fuzzy type t_f, the generic marker * has the following interpretation:*

*Let t_{fe} be the fuzzy type in extension associated with t_f and μ_e its membership function. The generic marker * is a fuzzy marker that associates the degree $\mu_e(t)$ with each individual marker m of I, where $t = \tau(m)$ and τ is the application from I to T_C, defined in the support, that attributes a minimum concept type to each individual marker.*

In Figure 4, the fuzzy type in extension t_{fe} associated with t_f is the fuzzy set that associates the degree 1 with *Whole milk* and its subtypes *Whole evaporated milk* and *Whole pasteurized milk*, 0.5 with *Half skim milk*, and 0 with the rest of T_C. The generic marker thus stands for the fuzzy set that associates the degree 1 with the markers of *Whole milk*, *Whole evaporated milk* and *Whole pasteurized milk*, 0.5 with the markers of *Half skim milk*, and 0 with the rest of I.

Remark 3 *The information brought by a generic marker, with the interpretation of Definition 7, is redundant with that given by its associated type t_f: the degree associated with a given individual marker in * is the same as the one associated with its type in t_f.*

Remark 4 *In the particular case where t_f is a "classic" type t, t_{fe} is the fuzzy type in extension that associates the value 1 with t and its subtypes, and the value 0 on the rest of T_C. The associated generic marker is thus the fuzzy marker that associates the value 1 with the markers of t and its subtypes, that is, with the markers that conform to t, which is in conformity with the Remark 1.*

3 The Specialization Relation for Fuzzy Conceptual Graphs

The specialization relation of the conceptual graph model allows one to perform comparisons of conceptual graphs. After having extended the model to represent fuzzy concepts (concepts with a fuzzy marker or with a fuzzy type), the next step is to preserve the specialization relation for conceptual graphs that include fuzzy concepts (called "fuzzy conceptual graphs"), in order to be able to compare fuzzy conceptual graphs, and particularly a fuzzy query graph with fuzzy factual graphs.

3.1 The notion of specialization for fuzzy sets

The notion of specialization for fuzzy sets is based on the inclusion relation: given two fuzzy sets A and B, A is a specialization of B if and only if A is included in B. An example is given in Figure 5 on a continuous domain. This definition applies to both discrete and continuous domains.

Definition 8 *Let A and B be two fuzzy sets defined on a domain X. A is included in B (denoted $A \subseteq B$) if and only if their membership functions μ_A and μ_B satisfy the condition:*

$$\forall x \in X, \mu_A(x) \leq \mu_B(x).$$

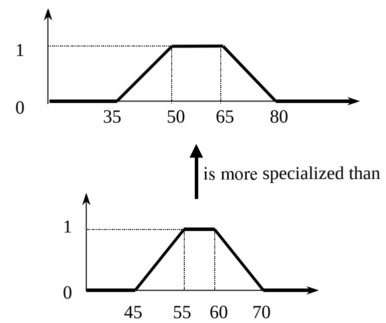

Fig. 5. Example of specialization for fuzzy sets

3.2 Extension of the specialization relation to fuzzy conceptual graphs

Definition 9 *Let m and m' be two markers of types t and t'. m and m' are thus defined on the same domain I. m' is a specialization of m if and only if $m' \subseteq m$, where $\subseteq$ is the classic inclusion relation defined for fuzzy sets.*

An example of specialization involving fuzzy markers is given in Figure 6.

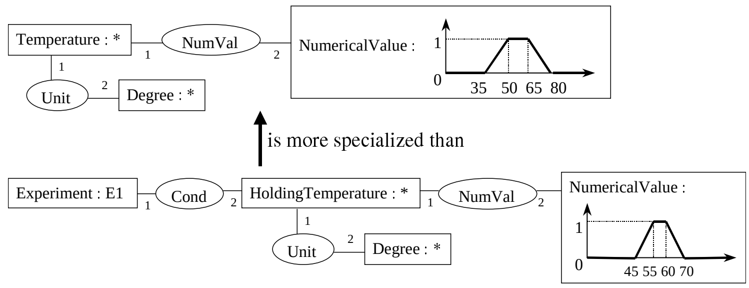

Fig. 6. An example of specialization involving fuzzy markers

Note that in Definition 9 there are four possible cases for m (resp. m'). m (resp. m') can be: an individual marker of a simple type; a fuzzy marker of a simple type; a generic marker of a simple type; a generic marker of a fuzzy type.

Remark 5 *In the "classic" conceptual graph model, there are three cases where m' is a specialization of m: (i) m and m' are individual and $m' = m$ (ii) $m = *$ and m' is individual (iii) $m = *$ and $m' = *$. In these cases, Definition 9 is in agreement with the classic specialization relation:*

If m and m' are two individual markers (of the simple types t and t', $t' \leq t$): m (resp. m') is represented by the fuzzy set that associates the value 1 with m (resp. m') and 0 with the rest of I, as specified in Remark 1. Then m' is a specialization of m iff $m' \subseteq m$, that is iff $m' = m$.

If m is the generic marker of a simple type t and m' an individual marker of a simple type t' ($t' \leq t$): m is represented by the fuzzy set that associates the value 1 with any individual marker that conforms to t (which includes m') and 0 with the rest of I, as specified in Remarks 1 and 4; m' is represented by the fuzzy set that associates the value 1 with m' and 0 with the rest of I. Then m' is a specialization of m because $m' \subseteq m$ is true.

If m is the generic marker of a simple type t and m' the generic marker of a simple type t' ($t' \leq t$): m is represented by the fuzzy set that associates the value 1 with any individual marker that conforms to t (which includes the individual markers that conform to t') and 0 with the rest of I; m' is represented by the fuzzy set that associates the value 1 with any individual marker that conforms to t' and 0 with the rest of I. Then m' is a specialization of m because $m' \subseteq m$ is true.

Definition 10 *Let t and t' be two types, t_{fe} and t'_{fe} their associated types in extension. t_{fe} and t'_{fe} are thus defined on the same domain T_C. t' is a specialization of t if and only if $t'_{fe} \subseteq t_{fe}$, where $\subseteq$ is the classic inclusion relation defined for fuzzy sets.*

An example of specialization involving fuzzy types is given in Figure 7.

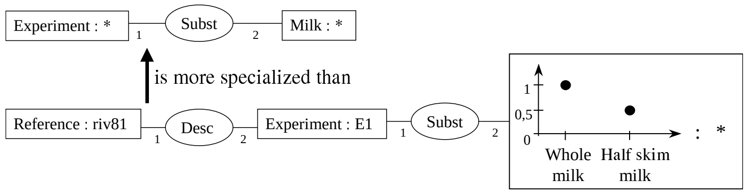

Fig. 7. An example of specialization involving fuzzy types

Remark 6 *In the "classic" case where t and t' are simple types, Definition 10 is in agreement with the classic specialization relation: t (resp. t') is represented by the fuzzy set that associates the value 1 with t (resp. t') and its subtypes, and 0 with the rest of T_C. Then t' is a specialization of t iff $t' \subseteq t$, that is iff the set* *of subtypes (including itself) of t' is included in the set of subtypes of t, that is iff $t' \leq t$.*

Definition 11 *Let $l = (t, m)$ and $l' = (t', m')$ be the labels of two concept vertices, where t and t' can be fuzzy types, m and m' can be fuzzy markers. Then l' is a specialization of l if and only if t' is a specialization of t and m' is a specialization of m.*

Definition 12 *The projection operation remains unchanged as a graph morphism that allows a restriction of the vertices labels [9], except that this restriction is now based on the specialization relation extended to fuzzy concepts as defined in Definitions 9 to 11.*

Using this extended projection operation, the comparison of two conceptual graphs leads to a binary result: a graph G can be projected into a graph G' or cannot, there is no intermediate solution. However, a more flexible comparison of fuzzy sets should allow one to evaluate the compatibility between two fuzzy conceptual graphs, and particularly between a fuzzy query graph and a fuzzy factual graph. Therefore, we propose to introduce a more flexible comparison that performs fuzzy querying.

4 A More Flexible Comparison of Fuzzy Conceptual Graphs

Two scalar measures are classically used in fuzzy set theory to evaluate the compatibility between a fuzzy selection criterion and a corresponding imprecise datum: (i) a possibility degree of matching [16]; (ii) a necessity degree of matching [5].

Definition 13 *Let m and m' be two markers of types t and t', with membership functions μ_m and $\mu_{m'}$. Then m' is compatible with m with the possibility degree $\Pi(m\ ; m')$ and the necessity degree $N(m\ ; m')$:*

- *$\Pi(m\ ; m')$, possibility degree of matching between m and m', is an "optimistic" degree of overlapping that measures the maximum compatibility between m and m', and is defined by:*
  $\Pi(m\ ; m') = sup_{x \in I} min(\mu_m(x), \mu_{m'}(x))$.
  sup denotes the supremum value of a function.
- *$N(m\ ; m')$, necessity degree of matching between m and m', is a "pessimistic" degree of inclusion that estimates the extent to which it is certain that m' is compatible with m. It is equal to the complement in 1 of the possibility degree of matching between the complement of m and m':*
  $N(m\ ; m') = 1 - sup_{x \in I} min(1 - \mu_m(x), \mu_{m'}(x))$.

Note that the measure of the possibility degree with which m' is compatible with m is symmetrical, whereas the necessity degree is not.

An example is given in Figure 8.

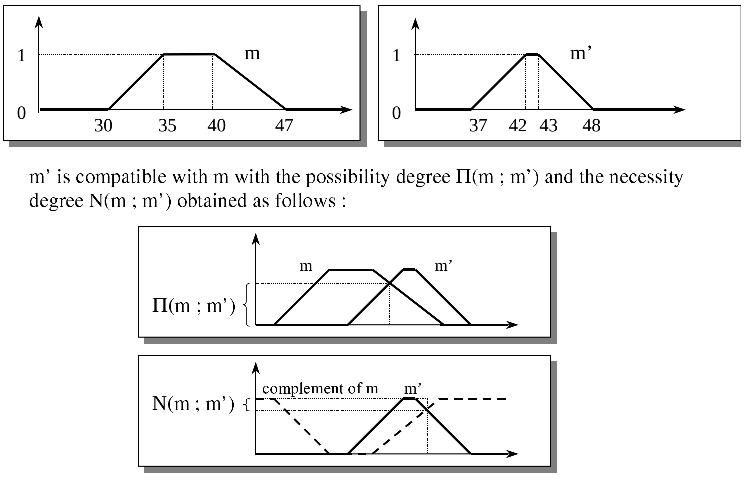

Fig. 8. Flexible comparison of two markers m and m'

Remark 7 *For the "classic" cases detailed in Remark 5, $\Pi(m\ ;m')$ and $N(m\ ;m')$ take the value 1.*

Definition 14 *Let t and t' be two types, t_{fe} and t'_{fe} their associated types in extension with membership functions μ_e and μ'_e. Then t' is compatible with t with the possibility degree $\Pi(t\ ;t')$ and the necessity degree $N(t\ ;t')$:*
$\Pi(t\ ;t') = sup_{x \in T_C} min(\mu_e(x), \mu'_e(x))$
$N(t\ ;t') = 1 - sup_{x \in T_C} min(1 - \mu_e(x), \mu'_e(x))$.

Remark 8 *For two "classic" types t and t': in the "classic" specialization case where $t' \leq t$, $\Pi(t\ ;t')$ and $N(t\ ;t')$ take the value 1. $\Pi(t\ ;t')$ takes the value 1 iff t and t' have common subtypes (in the broad sense). $N(t\ ;t')$ takes the value 1 iff $t' \leq t$.*

Definition 15 *Let $l = (t, m)$ and $l' = (t', m')$ be the labels of two concepts c and c', where t and t' can be fuzzy types, m and m' can be fuzzy markers. Then c' is compatible with c with the possibility degree $\Pi(c\ ;c')$ and the necessity degree $N(c\ ;c')$, where $\Pi(c\ ;c')$ and $N(c\ ;c')$ are defined as follows:*
$\Pi(c\ ;c') = min(\Pi(t\ ;t'), \Pi(m\ ;m'))$.
$N(c\ ;c') = min(N(t\ ;t'), N(m\ ;m'))$.

The *min* operator is used for the conjunction of the compatibility degrees, as presented in [6].

For instance, for two concepts c and c':
c = [*Whole milk : 1/sample32 + 0.5/sample35*] and
c' = [*0.8/Whole milk + 1/Half skim milk :* *], we have:
$\Pi(t\ ;t') = 0.8$ (given by *Whole milk* which has the degree 1 in c and 0.8 in c'),

$\Pi(m\ ; m') = 0.8$ (both 'sample32' and 'sample35', which are individual markers of type *Whole milk*, have the degree 0.8 in c', where the generic marker $*$ stands for the fuzzy set that associates the degree 0.8 with every marker that conforms to *Whole milk* and 1 with every marker that conforms to *Half skim milk*; the result is due to 'sample32' which has the degree 1 in c),
$N(t\ ; t') = 0$ (*Half skim milk* is indeed fully possible in c' whereas it does not belong to the type of c),
$N(m\ ; m') = 0$ (for the same reason: the markers that conform to *Half skim milk* are fully possible in c' whereas they do not belong to the marker of c),
$\Pi(c\ ; c') = min(0.8, 0.8) = 0.8$,
$N(c\ ; c') = min(0, 0) = 0$.

Definition 16 *Let G and G' be two conceptual graphs that can possibly include fuzzy concepts. Then the graph G' is compatible with the graph G with the possibility degree $\Pi(G\ ; G')$ and the necessity degree $N(G\ ; G')$ if there is an ordered pair (f, g) of mappings, f (resp. g) from the set of relation types (resp. concept types) of G to the set of relation types (resp. concept types) of G', such that:*

- *the edges and their numbering are preserved;*
- *the relation vertex labels may be restricted.*

$\Pi(G\ ; G')$ and $N(G\ ; G')$ are then determined as follows:

Let C_G be the set of concept vertices of G. For each concept vertex $c \in C_G$, let $\Pi(c\ ; g(c))$ be the possibility degree and $N(c\ ; g(c))$ the necessity degree with which $g(c)$ is compatible with c. Then $\Pi(G\ ; G') = min_{c \in C_G} \Pi(c\ ; g(c))$ and $N(G\ ; G') = min_{c \in C_G} N(c\ ; g(c))$

For example, let us consider the graph G given in Figure 9 and the graph G' given in Figure 10: G' is compatible with G with the possibility degree 0.8. This value corresponds to the possibility degree with which the concept with a fuzzy marker of G' (representing the numerical interval [4;6]) is compatible with the concept with a fuzzy marker of G (of type *NumericalValue*). All the other concept vertices of G' are compatible with the possibility degree 1 with their antecedent in G (including the concept vertex [*Milk* : *]). G' is compatible with G with the necessity degree 0, which is obtained for the [*Milk* : *] vertex.

Remark 9 *In the case of a "classic" projection operation (G and G' are not fuzzy, and G can be projected into G'), $\Pi(G\ ; G')$ and $N(G\ ; G')$ take the value 1. The opposite is not true. We can also note that the extended projection operation presented in Definition 12 is more restrictive than a graph compatibility with the possibility degree 1, but less restrictive than a graph compatibility with the necessity degree 1.*

5 Conclusion and Perspectives

In this paper, the conceptual graph model has been extended in order to be able to (i) represent imprecise data as well as preferences in queries (ii) propose the

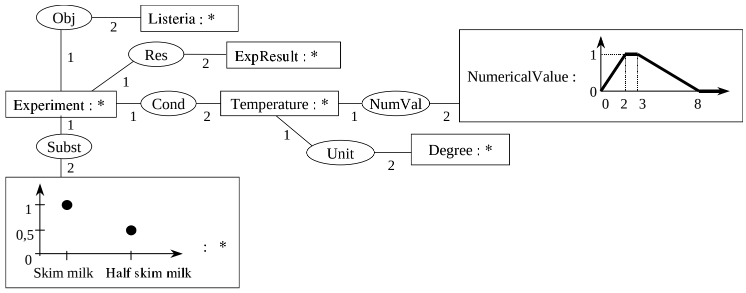

Fig. 9. An example of a query graph G

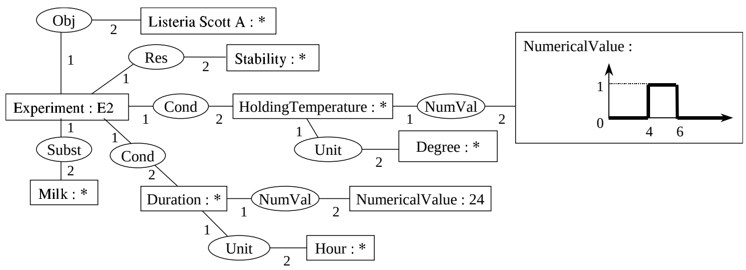

Fig. 10. An example of a factual graph G'

extension of the projection operation in order to take into account fuzzy values (iii) perform a softer comparison using the possibility degree of matching and the necessity degree of matching between two graphs.

This work is part of a food risk control application which uses two databases: a relational database that has been well experimented, and the conceptual graph base mentioned in this article. The latter has been implemented using the CoGITaNT platform [7], including all the mechanisms presented in this paper and the unified querying system presented in [2]. It has been successfully presented to our microbiologist partners and is now operational. At the present time, it contains more than a hundred graphs and appears as an interesting and useful complement to the relational database. Conceptual graphs are drawn through the graphic interface of CoGITaNT, and registered as text files.

An important progress in our work has been the definition of the fuzzy type in extension, presented in this paper. This notion allowed us to deal with fuzzy types that are defined on the same domain for all of them and thus to be able to apply results, namely comparison degrees, from the fuzzy set theory.

Our next work will be focused on three different points: (i) the logical interpretation of the conceptual graph model extended to fuzzy values, (ii) the notion of equivalence classes for fuzzy types in intention that are associated with the same fuzzy type in extension, (iii) the study of the complexity of the model extended to fuzzy values, which has already been approached in [12].

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