Logical Interpretation of Fuzzy Conceptual Graphs

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Abstract

In previous studies, we have extended the conceptual graph model, which is a knowledge representation model belonging to the family of semantic networks, to be able to represent fuzzy values. The basic conceptual graph model has a logical interpretation in first-order logic. In this paper, we focus on the logical interpretation of the conceptual graph model extended to fuzzy values: we use logical implications stemming from fuzzy logic, so as to extend the logical interpretation of the model to fuzzy values and to comparisons between fuzzy conceptual graphs.

1. Introduction

Our research takes place in the framework of a national project that aims at building a tool for the analysis of microbial risks in food products. In this scope, a relational database has been built in order to store information from microbiological publications. It is complemented by a conceptual graph knowledge base [1], which is used to store information that do not conform to the structure of the relational database (we call them weakly structured data in the following). The data stored in both bases may be imprecise, then represented by possibility distributions. Moreover, they are consulted using queries that express the user's preference levels, by means of fuzzy sets [2]. One of the reasons of this choice is to ensure compatibility and uniformity between the representations of both imprecise data and flexible queries.

Therefore, we have proposed a way of representing fuzzy values, and introduced fuzzy mechanisms in both bases. In particular, we have extended the conceptual graph model to take into account fuzzy values [3, 4]. In this paper, we focus on a particular aspect of this extended model: its logical interpretation, which is one of the strong points of the model. The basic conceptual graph model has a logical interpretation in first-order logic. Our purpose is to extend this logical interpretation to take into account fuzzy values.

In Section 2, we briefly present the classic conceptual graph model. In Section 3, we recall our choices for representing fuzzy values in the conceptual graph model and for comparing fuzzy conceptual graphs. In Section 4, which involves the main contribution of the paper, we propose to extend the logical interpretation of the conceptual graph model to take into account fuzzy values and comparisons between fuzzy conceptual graphs.

2. The classic conceptual graph model

The weakly structured data of the application are represented using the conceptual graph model, which is a knowledge representation model based on labelled graphs, introduced by Sowa [1]. In the following, and more generally in our work, we use the formalization presented in [5].

In the conceptual graph model, knowledge is divided into two parts: a terminological part, called the support, and an assertional part represented by the conceptual graphs, which are labelled graphs. In this section, we briefly and intuitively present the conceptual graph model through the example of our application.

2.1. The support

The support provides the ground vocabulary used to build the knowledge base: the types of concepts used, the instances of these types, and the types of relations linking the concepts. It also describes the hierarchical organization of these elements.

The set of concept types is partially ordered by the "a kind of" relation. *Universal* and *Absurd* are respectively its greatest and lowest elements. Figure 1 presents a part of the set of concept types used in the application. The information stored in the application describes the behaviour of pathogenic germs (increase, decrease or stability of the concentration of pathogens such as Listeria for instance) in food products, during food transformation processes (heating, cutting, storage, mixing, etc.).

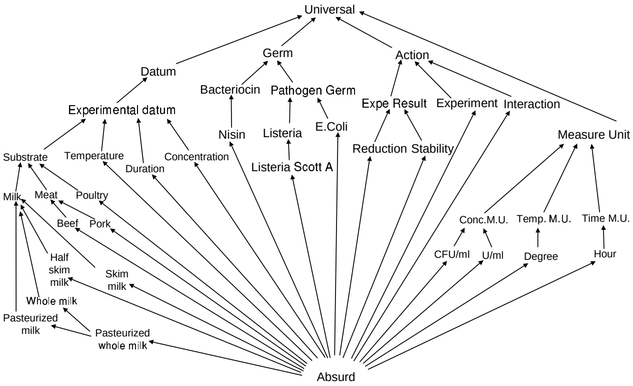

Figure 1. A part of the concept type set for the microbial application

The concepts can be linked by means of relations. The set of relation types is also partially ordered by the "a kind of" relation. Each relation type is characterized by an arity, and a signature which specifies the maximal concept types that a given relation can link together. The set of relation types we use contains relation types such as *Agt* (Agent), which is a binary relation having *(Action, Germ)* as a signature. It means that "an action has for agent a germ" (for example, an interaction can have a bacterium as an agent).

The third set of the support is the set of individual markers. Each individual marker represents an instance of a concept. For example, *Celsius degree* can be an instance of *Degree*. The generic marker (noted $*$) is a particular marker referring to an unspecified instance of a concept.

2.2. The conceptual graphs

The conceptual graphs, built upon the support, express the factual knowledge. They are composed of two kinds of vertices: (i) the concept vertices (noted in rectangles), which represent the entities, attributes, states, events; (ii) the relation vertices (noted in ovals), which express the nature of the relations between concepts.

The label of a concept vertex is a pair defined by the type of the concept and a marker (individual or generic) of this type. The label of a relation vertex is its relation type.

For example, the conceptual graph given in Figure 2 is a representation of the information: "the experiment E1 carries out an interaction I1 between Nisin and Listeria Scott A in skim milk and the result is reduction".

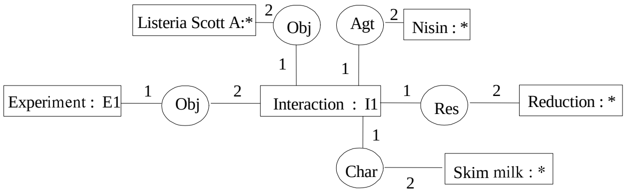

Figure 2. An example of a conceptual graph

2.3. Specialization relation, projection operation

The set of conceptual graphs is partially ordered by the specialization relation (noted $\leq$), which can be computed by the projection operation (a kind of graph morphism allowing a restriction of the vertex labels authorized in the support): $G' \leq G$ if and only if there is a projection of G into G'. An example is given in Figure 3.

Since it allows the search for conceptual graphs which are specializations of (which contain more precise information than) another conceptual graph, the projection operation is widely used for the querying of conceptual graph knowledge bases. We then call a "query graph" a conceptual graph that we try to project into each graph of the knowledge base, called "factual graphs".

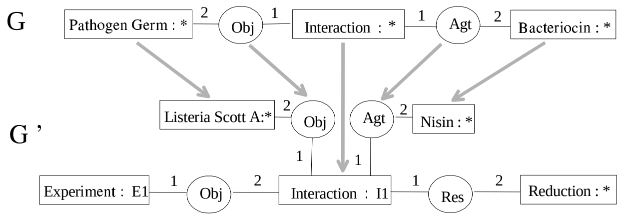

Figure 3. There is a projection from G into G', $G' \leq G$ (G' is a specialization of G). The meaning of the arrows is: "can be projected into".

2.4. Logical interpretation of the classic conceptual graph model

In the classic conceptual graph model, the support, the conceptual graphs, and the specialization relation, have a logical interpretation in first-order logic.

The support of the conceptual graph model has a logical interpretation, represented by several first-order logic formulas. In particular, the *kind of* relation between types is represented by a logical implication. For instance, any individual marker of type “Skim milk” is also a kind of “Milk”. This is translated by the following formula:

$\forall t_1, t_2 \in T_C, such\ that\ t_2 < t_1 : \forall x\ (t_2(x) \rightarrow t_1(x)).$

Each conceptual graph also has a logical interpretation which is a first-order logic formula. In this formula, each generic marker is associated with a distinct variable, each individual marker with a constant, each concept type with a unary predicate applied to its marker, each relation type with a predicate applied to the markers of the concept vertices it links. The formula associated with the conceptual graph is then the existential closure of the conjunction of all atoms. For instance, the logical interpretation of the conceptual graph represented in Figure 2, is the following :

$\exists x, y, z, t \quad (ListeriaScottA(x) \wedge Nisin(y) \wedge Experiment(E1) \wedge Interaction(I1) \wedge Reduction(z) \wedge SkimMilk(t) \wedge Obj(I1, x) \wedge Agt(I1, y) \wedge Obj(E1, I1) \wedge Res(I1, z) \wedge Char(I1, t)).$

The specialization relation, and thus the projection operation, between two graphs, is equivalent to the logical implication between their corresponding formulas, provided that they are in normal form [5]: each individual marker appears at most once in the same graph. This equivalence is due to the fact that, given two conceptual graphs G and H, H more specific than G means that, in the formula associated with G, it is possible to find a substitution [6] of each term or atom by a more specific one (with the meaning of the logical implication), that appears in the formula associated with H. For example, in Figure 3, *Listeria Scott A* is a substitution of *Pathogen Germ*, *I1* is a substitution of *, etc.

3. The conceptual graph model extended to fuzzy values

In our previous works [3, 4], we have introduced fuzzy values in the conceptual graph model. We briefly present them in Section 3.1. Then, we have proposed two different approaches to compare conceptual graphs that possibly include fuzzy values. The first approach, presented in Section 3.2, is an all-or-nothing process based on the notion of inclusion. The second approach, presented in Section 3.3, is a more flexible process based on the notion of graded matching.

3.1. Fuzzy markers, fuzzy types

We have introduced the notion of fuzzy type, which is a fuzzy set defined on the concept type set of the support, and the notion of fuzzy marker, which is a fuzzy set defined on the set of markers of the support.

The conceptual graph represented in Figure 4 includes a concept with a fuzzy marker, of type *NumericalValue*.

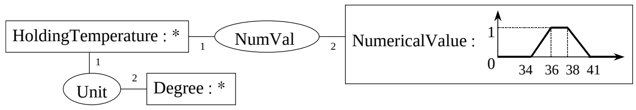

Figure 4. An example of a concept with a fuzzy marker

Figure 5 gives an example of a concept with a fuzzy type: its concept type is a fuzzy set defined on the concept type set of the support, and its marker is generic.

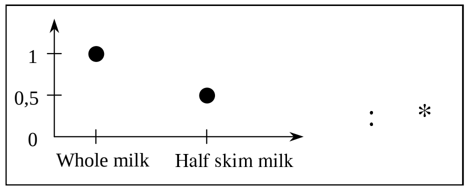

Figure 5. An example of a concept with a fuzzy type

3.2. Extended specialization relation

We then have redefined the specialization relation in order to take into account the fuzzy types and fuzzy markers introduced. The notion of specialization for fuzzy sets is based on the inclusion relation. Given two fuzzy sets A and B, A is a specialization of B if and only if A is included in B: the membership function of A is under the membership function of B on the whole domain.

An example of specialization involving fuzzy markers is given in Figure 6.

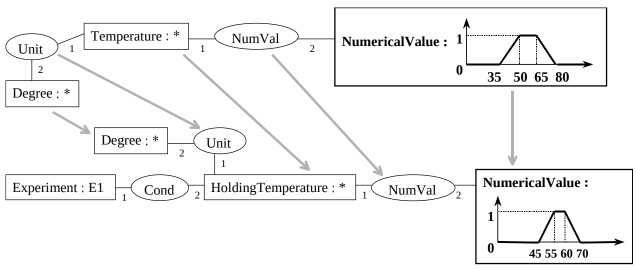

Figure 6. An example of specialization involving fuzzy markers

3.3. A more flexible comparison operation characterized by matching degrees

We finally have relaxed the all-or-nothing projection operation by defining a more flexible comparison operation characterized by two compatibility degrees: the possibility degree of matching and the necessity degree of matching, that we use here to compare two graphs. Given two fuzzy sets A and B defined on a domain X, with membership functions μ_A and μ_B, B is compatible with A with the possibility degree $\Pi(A\ ; B)$ [7] and the necessity degree $N(A\ ; B)$ [8] given by:

$$\Pi(A\,; B) = sup_{x \in X} min(\mu_A(x), \mu_B(x)).$$

$$N(A\,; B) = 1 - sup_{x \in X} min(1 - \mu_A(x), \mu_B(x)) \\ = inf_{x \in X} max(1 - \mu_A(x), \mu_B(x))).$$

An example is given in Figure 7.

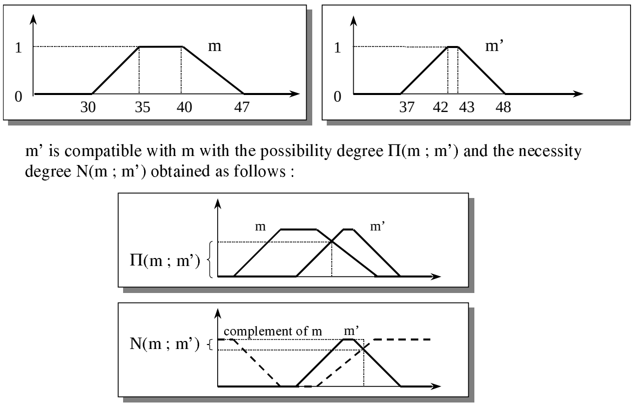

Figure 7. Flexible comparison of two markers m and m'

The min operator is used for the conjunction [9] of the compatibility degrees of all the elements of the graphs.

4. Logical interpretation of the conceptual graph model extended to fuzzy values

In this part, we present the impact of the introduction of fuzzy values, on the logical interpretation of the model. Two issues have to be considered: (i) how to represent fuzzy markers and fuzzy types in the logical formula associated with a conceptual graph ? (ii) can the extended specialization relation and the flexible comparison operation still be expressed as logical implications ?

4.1. Logical interpretation of the fuzzy conceptual graphs

In the conceptual graph model extended to fuzzy values, we have to take into account the logical interpretation of two new cases: fuzzy markers and fuzzy types. Compared to the classic interpretation, predicates associated with fuzzy types, and constants associated with fuzzy markers, now need to be extended.

Definition 1 *The logical interpretation of a fuzzy type is composed of two parts:*

- *a unary predicate P in first-order logic;*
- *an interpretation of P as a fuzzy set defined on the predicates associated with the types of the support.*

For instance, the fuzzy type of Figure 5 is associated with the predicate $UnskimmedMilk$. The conceptual graph of Figure 5, which is reduced to a single concept, has the following logical interpretation:

$$\begin{cases} \exists x\ UnskimmedMilk(x) \\ UnskimmedMilk = \begin{array}{l} 1/WholeMilk \\ +0.5/HalfSkimMilk \end{array} \end{cases} \quad (1)$$

Remark 1 *A classic concept type t can be considered as a particular case of a fuzzy type: its associated predicate is the fuzzy set defined on one element, the predicate t (with the degree 1).*

Definition 2 *The logical interpretation of a fuzzy marker is composed of two parts:*

- *a constant c;*
- *an interpretation of c as a fuzzy set defined on the constants associated with the markers of the support.*

For instance, the fuzzy marker of Figure 4 is associated with the constant $HumanBodyValue$. The conceptual graph of Figure 4 has the following logical interpretation:

$$\begin{cases} \exists y, z\ (HoldingTemperature(y) \wedge \\ NumericalValue(HumanBodyValue) \wedge Degree(z) \\ \wedge\ NumVal(y, HumanBodyValue)\ \wedge\ Unit(y, z)) \\ \\ HumanBodyValue = \int_{[-\infty,34]} 0/x\ + \\ \int_{[34,36]} (0.5x - 17)/x\ +\ \int_{[36,38]} 1/x\ + \\ \int_{[38,40]} (-0.33x + 13.67)/x\ +\ \int_{[40,\infty]} 0/x \end{cases} \quad (2)$$

The notations used in (1) and (2) indicate the degree associated with each element in a fuzzy set, respectively for a discrete and a continuous domain.

Remark 2 *A classic individual marker m can be considered as a particular case of a fuzzy marker: its associated constant is the fuzzy set that associates the value 1 with the constant m and 0 elsewhere.*

In the following, we focus on the logical interpretation of the two approaches we have proposed to compare conceptual graphs that possibly include fuzzy values. As we will see in Sections 4.2 and 4.3 respectively, the first approach (an all-or-nothing process based on the notion of inclusion) can be handled using one implication degree, whereas the second approach (a more flexible process based on the notion of graded matching) can be handled using two other implication degrees.

4.2. Logical interpretation of the extended specialization relation

We have to show that the extended specialization relation that we have introduced for fuzzy values has a logical foundation, that is, given two conceptual graphs G and H, H more specific than G still means that, in the formula associated with G, it is possible to find a substitution of each term or atom by a more specific one (with the meaning of the logical implication [10]), that appears in the formula associated with H.

In order to decide if a predicate in the formula associated with G, called $Predicate1$, may be substituted by a predicate in the formula associated with H, called $Predicate2$, we must know if $Predicate2$ implies $Predicate1$ with the meaning of a logical implication. In the same way, in order to determine if a given marker $m2$ in H is more specific than a marker $m1$ in G, we must know if $m2$ implies $m1$ with the meaning of a logical implication.

In this part, we are going to explain why this extended specialization relation corresponds to Rescher-Gaines' implication, which extends the classic implication.

In our definition of the specialization relation extended to fuzzy values, $Predicate2$ is said to be more specific than $Predicate1$, if and only if $Predicate2$ is included in $Predicate1$, with the meaning of the fuzzy sets inclusion relation. Let μ_1 and μ_2 be the respective membership functions of $Predicate1$ and $Predicate2$. We must examine if μ_2 is under μ_1 in every point of the domain. For a given point x of the domain, the truth value of "μ_2 is under μ_1", is precisely the result given by Rescher-Gaines' implication, which takes the value 1 if $\mu_2(x)$ is smaller than $\mu_1(x)$, and 0 otherwise. On the whole domain, "$Predicate2$ is included in $Predicate1$" is true if and only if μ_2 is under μ_1 in every point, that is, if Rescher-Gaines' implication takes the value 1 in every point. The infimum of Rescher-Gaines' implication on the whole domain (or Rescher-Gaines' implication degree), must be 1.

Deciding if $Predicate2$ is more specific than $Predicate1$, in the extended specialization relation we have defined, is equivalent to evaluating Rescher-Gaines' implication degree between $Predicate2$ and $Predicate1$, which is equal to 0 or 1. The same reasoning also applies to markers.

We can now redefine the substitution mechanism in order to extend it to the predicates and constants that are respectively associated with fuzzy types and fuzzy markers.

Definition 3 *A predicate $P2$ (resp. a constant $c2$) is a possible substitution of a predicate $P1$ (resp. a constant $c1$) if and only if the interpretation of $P2$ (resp. $c2$) as a fuzzy set implies the interpretation of $P1$ (resp. $c1$) as a fuzzy set, with the meaning of Rescher-Gaines' implication.*

For example, the conceptual graphs of Figure 6 have the following logical interpretations:

$$\begin{cases} \exists y, z\ (Temperature(y) \wedge \\ NumericalValue(RequestedValue) \wedge Degree(z) \\ \wedge NumVal(y, RequestedValue) \wedge Unit(y, z)) \\ \\ RequestedValue = \int_{[-\infty,35]} 0/x \ + \\ \int_{[35,50]} (0.07x - 2.33)/x \ + \ \int_{[50,65]} 1/x \ + \\ \int_{[65,80]} (-0.07x + 5.33)/x \ + \ \int_{[80,\infty]} 0/x \end{cases}$$

$$\begin{cases} \exists t, u\ (Experiment(E1) \wedge HoldingTemperature(t) \\ \wedge NumericalValue(ObservedValue) \wedge Degree(u) \\ \wedge Cond(E1, t) \wedge NumVal(t, ObservedValue) \wedge \\ Unit(t, u)) \\ \\ ObservedValue = \int_{[-\infty,45]} 0/x \ + \\ \int_{[45,55]} (0.1x - 4.5)/x \ + \ \int_{[55,60]} 1/x \ + \\ \int_{[60,70]} (-0.1x + 7)/x \ + \ \int_{[70,\infty]} 0/x \end{cases}$$

The substitutions corresponding to the projection of the first graph into the second one are the following: {(Temperature, HoldingTemperature), (y, t), (NumericalValue, NumericalValue), (RequestedValue, ObservedValue), (Degree, Degree), (z, u), (NumVal, NumVal), (Unit, Unit)}. $RequestedValue$ can be substituted by $ObservedValue$ because the fuzzy set interpretation of $ObservedValue$ implies the fuzzy set interpretation of $RequestedValue$ with the meaning of Rescher-Gaines' implication.

4.3. Logical interpretation of the flexible comparison operation characterized by matching degrees

In order to relax the all-or-nothing specialization relation, we use the possibility degree of matching and the necessity degree of matching. In this part, we aim at showing that the comparison between two graphs using the necessity degree of matching is equivalent to Kleene-Dienes' implication. The one using the possibility degree of matching is related to Mamdani's implication, which does not extend the classic implication.

For a given point x of the domain, Kleene-Dienes' truth value of "$Predicate2$ implies $Predicate1$", is defined as $max(1 - \mu_1(x), \mu_2(x))$. On the whole domain, the degree of “$Predicate2$ implies $Predicate1$” according to Kleene-Dienes’ implication, is the infimum of the values obtained for every point of the domain using Kleene-Dienes’ implication. This result is exactly the necessity degree of matching between $Predicate1$ and $Predicate2$. The same reasoning also applies to markers.

Definition 4 *A predicate $P2$ (resp. a constant $c2$) is said to be a possible substitution of a predicate $P1$ (resp. a constant $c1$) with the necessity degree $d = N(P1\,;P2)$ (resp. $N(c1\,;c2)$) if and only if Kleene-Dienes’ implication degree of $P2 \rightarrow P1$ (resp. $c2 \rightarrow c1$) is equal to d.*

Mamdani’s truth value of “$Predicate2$ implies $Predicate1$”, is defined as $min(\mu_1(x), \mu_2(x))$, for any point x of the domain. If we take the supremum of the values obtained on the whole domain (instead of the infimum chosen in the previous cases), which is an optimistic choice for the aggregation of the results on the whole domain, we obtain the possibility degree of matching between $Predicate1$ and $Predicate2$. The same reasoning also applies to markers.

Remark 3 *The possibility degree has two limits: (i) it is not based on a logical implication, as Mamdani’s implication does not extend the classic implication; (ii) it does not consider the infimum value obtained on the domain, which corresponds to a conjunctive aggregation, as used in implication degrees, but the supremum. On the contrary, the necessity degree is an implication degree and fits into the extension of the logical interpretation of the conceptual graph model in a natural way.*

Definition 5 *A predicate $P2$ (resp. a constant $c2$) is said to be a possible substitution of a predicate $P1$ (resp. a constant $c1$) with the possibility degree $d = \Pi(P1\,;P2)$ (resp. $\Pi(c1\,;c2)$) if and only if the supremum of Mamdani’s implication of $P2 \rightarrow P1$ (resp. $c2 \rightarrow c1$) is equal to d.*

For example, let us consider the first conceptual graph of Figure 6, noted G, and the conceptual graph of Figure 4, noted H. The possible substitutions from G to H and their respective necessity degrees are the following: {(Temperature, HoldingTemperature, 1), (y, y, 1), (NumericalValue, NumericalValue, 1), (RequestedValue, HumanBodyValue, 0.12), (Degree, Degree, 1), (z, z, 1), (NumVal, NumVal, 1), (Unit, Unit, 1)}. Their possibility degrees are the following: {(Temperature, HoldingTemperature, 1), (y, y, 1), (NumericalValue, NumericalValue, 1), (RequestedValue, HumanBodyValue, 0.33), (Degree, Degree, 1), (z, z, 1), (NumVal, NumVal, 1), (Unit, Unit, 1)}.

The min operator is then used for the conjunction of the compatibility degrees on the whole graphs: H is compatible with G with the possibility degree 0.33 and the necessity degree 0.12.

5. Conclusion

In this paper, we have focused on the logical interpretation of the conceptual graph model extended to fuzzy values, that we have developped in previous studies. We have proposed a way of integrating fuzzy types and fuzzy markers in the logical formula associated with a graph. Then, we have shown that the specialization relation extended to fuzzy values that we have introduced corresponds to Rescher-Gaines’ implication, which extends the classic implication. Finally, we have shown that the comparison between two graphs using the necessity degree of matching is based on Kleene-Dienes’ implication. The one using the possibility degree of matching is related to Mamdani’s implication.

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